

## Főkomponens analízis

Mérési adatok  $\rightarrow$  dimenziós vektor

$\phi_i$  ortonormális ( $\phi_i \phi_j = \delta_{ij}$ ) bázis:

$$\mathbf{X} = \sum_{i=1}^n \mathbf{y}_i \phi_i = \Phi \mathbf{Y}$$

itt  $\Phi = [\phi_1 \dots \phi_n]$  és  $\mathbf{Y} = [\mathbf{y}_1 \dots \mathbf{y}_n]^T$ .

Az ortonormalitás miatt  $\mathbf{y}_i = \phi_i \mathbf{X}$

$\mathbf{Y} \mathbf{X}$  elforgatottja

$\phi_i$ : tulajdonság,  $\mathbf{y}_i$  méri

Keressük  $m (< n)$  olyan  $\phi$ -t, amelyik  $\mathbf{X}$ -et legjobban közelíti!

$\mathbf{Y}$  nem használt tagjait (előre meghatározandó)  $b_i$  konstansokkal helyettesítjük:

$$\hat{\mathbf{X}}(m) = \sum_{i=1}^m \mathbf{y}_i \phi_i + \sum_{i=m+1}^n b_i \phi_i$$

Minimalizálás  $\chi^2$  eltérés-négyzet

$$\begin{aligned}
 \chi^2 &= E\|\mathbf{X} - \hat{\mathbf{X}}(m)\| \\
 &= E \sum_{i=m+1}^n \sum_{j=m+1}^n (\mathbf{y}_i - b_i)(\mathbf{y}_j - b_j) \phi_i^T \phi_j \\
 &= \sum_{i=m+1}^n E(\mathbf{y}_i - b_i)^2
 \end{aligned}$$

$E()$ : várható érték operátor

Minimum: deriválni kell  $\chi^2$ -t  $b_i$  szerint!

$$b_i = E\mathbf{y}_i.$$

Ezt visszaírva  $\chi^2$ -be:

$$\begin{aligned}
 \chi^2 &= \sum_{i=m+1}^n E(\mathbf{y}_i - E[\mathbf{y}_i])^2 \\
 &= \sum_{i=m+1}^n \phi_i^T E(\mathbf{X} - E[\mathbf{X}])(\mathbf{X} - E[\mathbf{X}])^T \phi_i \\
 &= \sum_{i=m+1}^n \phi_i^T \Sigma_X \phi_i
 \end{aligned}$$

$\Sigma_X$  az adatok kovariancia mátrixa!

Bebizonyítható, hogy  $\phi_i$ -re az optimum

$$\Sigma_X \phi_i = \lambda_i \phi_i$$

$\phi_i$  a  $\lambda_i$  sajátértékhez tartozó sajátvektor

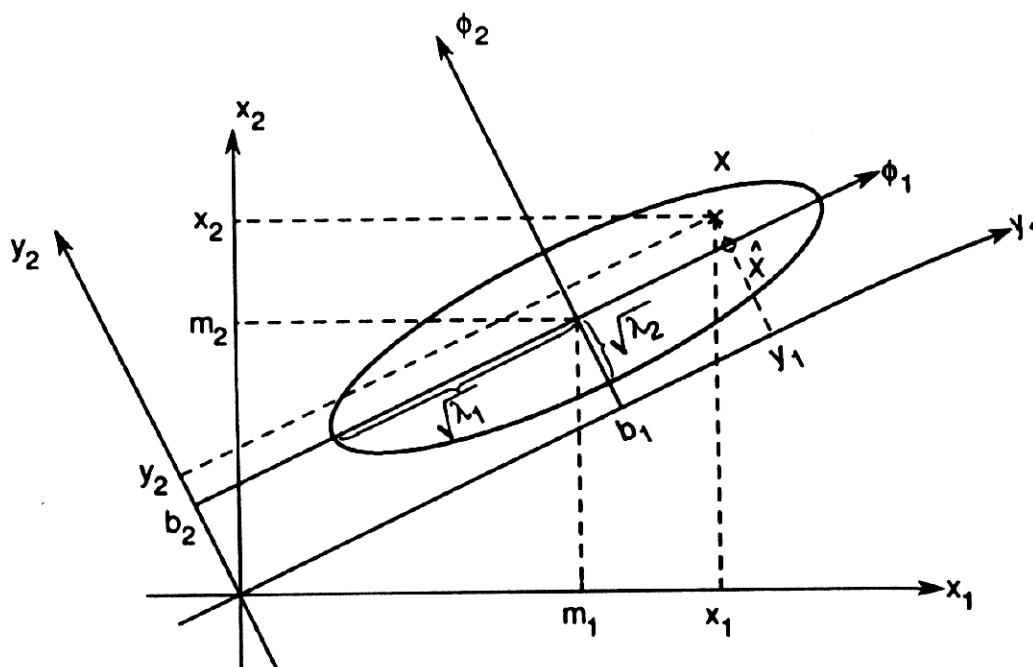
$$\chi^2 = \sum_{i=m+1}^n \lambda_i$$

Azt az eredményt kaptuk tehát, hogy a (a  $\chi^2$  közelítés értelemben) Legjobb lineáris reprezentáció:

kovariancia mátrix sajátvektorai szerinti ortogonális transzformáció alapján!

Ha  $i$  monoton csökken,  $\lambda_i \geq \lambda_j$ , ha  $i > j$

$m (< n)$  „fő” komponens minimalizálja az eltérést!



A  $\phi_1$  és  $\phi_2$  sajátvektorok az eloszlás fő tengelyei

$\lambda_1$  és  $\lambda_2$  sajátértékek:

$\phi_1$  és  $\phi_2$  mentén az eloszlás varianciája

Mivel  $y_i = \phi_i^T \mathbf{X}$ ,

ezért  $y_1$  és  $y_2$  lesz  $\mathbf{X}$  vetületei  $\phi_1$  és  $\phi_2$  tengelyekre.

A  $y_i$  tulajdonságok

ha töröljük az  $y_i$  tulajdonságot, akkor a közelítés hibája  $\lambda_i$ -vel nő meg.

Veszteséges tömörítés:

csak a legnagyobb  $m$  sajátkomponenst és az arra vett vetületek alapján!

Visszaállítás: átlagos eltérés értéke

$$\sum_{i=m+1}^n \lambda_i$$

ha lesz. Ha ez sokkal kisebb, mint

$$\sum_{i=1}^m \lambda_i$$

, és  $m$  sokkal kisebb, mint  $n$ , akkor jelentős tömörítés!

Egyes tulajdonságok egymástól függetlenek:

$y_i$  egymás közötti korrelációja 0.

Adatok entrópiájára is szélsőérték:

az összes lineáris transzformáció közül ez a transzformáció minimalizálja a transzformációk  $Y$  terében mért entrópiamaximumot (*minimax* viselkedés)!

Stacionárius idősorok + főkomponens analízis

$y_i$  tulajdonság-függvényei  $e^{j\omega_i t}$  alakúak !  
Visszakapjuk a Fourier-transzformációt!

Főkomponens analízis hátrányai:  
nem mindig (fizikailag) értelmes levonni  $E[\mathbf{X}]$ -t

Csak lineáris tulajdonságok! Ellenpélda: adott síkban körívet leíró adatok

Ezt a hátrányt az adatok Normalizálás:

$\mathbf{x}_i$  adatok  $\|\mathbf{x}\|$  szerint normálva

a főkomponens analízis rendben végrehajtható!

$\mathbf{z}_i = \mathbf{x}_i / \sum_{j=1}^n \mathbf{x}_j$  normálás

$\Sigma_X$  szinguláris!

## Tömörítési eljárások

jpeg, gif, GSM, compress, pkzip, (arc, rar ...) redundáns adatok:  
tömörítés sikeres!

Veszteség nélküli és veszteséges eljárások

Futási hossz:

Pl. a 127 alatti karakter: darabszám a következő ismétlődő karakterre

128 feletti karakter: x-128 darab különböző karakter

Nem mindig hatásos: pl. egy ABABAB...

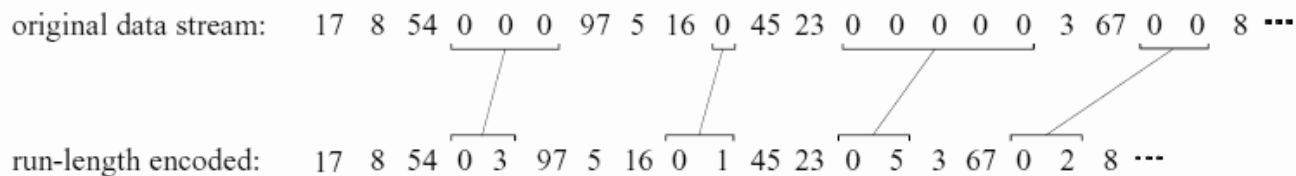


FIGURE 27-1

Example of run-length encoding. Each run of zeros is replaced by two characters in the compressed file: a zero to indicate that compression is occurring, followed by the number of zeros in the run.

Delta kódolás:

csak a különbség: kevesebb bit!

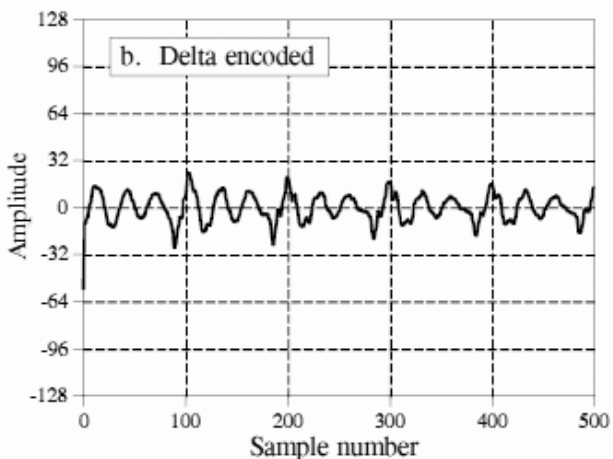
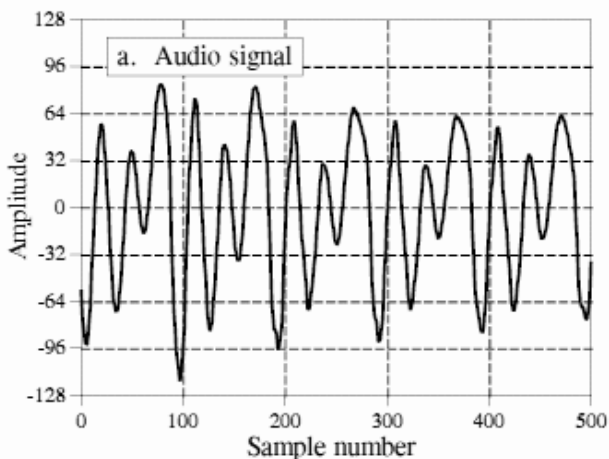
original data stream: 17 19 24 24 24 21 15 10 89 95 96 96 96 95 94 94 95 93 90 87 86 86 ...

$\begin{array}{c} \text{move} \\ \downarrow \\ \text{delta} \\ \downarrow \\ \text{delta} \\ \downarrow \\ \text{delta} \\ \downarrow \\ \text{delta} \\ \downarrow \\ \dots \end{array}$

delta encoded: 17 2 5 0 0 -3 -6 -5 79 6 1 0 0 -1 -1 0 1 -2 -3 -3 -1 0 ...

FIGURE 27-4

Example of delta encoding. The first value in the encoded file is the same as the first value in the original file. Thereafter, each sample in the encoded file is the difference between the current and last sample in the original file.



## Lineáris Prediktív Kódolás (LPC)



Lempel-Ziv-Welch eljárás alapötlet: szótár létrehozása

| stringtábla |    | szótár |    |
|-------------|----|--------|----|
| a           | 1  | a      | 1  |
| b           | 2  | b      | 2  |
| c           | 3  | c      | 3  |
| -----       |    | -----  |    |
| ab          | 4  | 1b     | 4  |
| ba          | 5  | 2a     | 5  |
| abc         | 6  | 4c     | 6  |
| cb          | 7  | 3b     | 7  |
| bab         | 8  | 5b     | 8  |
| baba        | 9  | 8a     | 9  |
| aa          | 10 | 1a     | 10 |
| aa          | 11 | 10a    | 11 |
| aaaa        | 12 | 11a    | 12 |

- minden bejövő adat: új szó a maximális egyezésű szótárelemmel
- tömörített kódot a maximális egyezésű szótárelem
- szótár mérete tipikusan  $2^{12}$ - $2^{16}$
- ha szótár megtelik → letörlik (csak az induló szótár marad meg ASCII!)

|                        |          |          |          |          |          |           |   |           |           |           |   |   |   |   |   |   |
|------------------------|----------|----------|----------|----------|----------|-----------|---|-----------|-----------|-----------|---|---|---|---|---|---|
| <b>bemenő<br/>adat</b> | a        | b        | a        | b        | c        | b         | a | b         | a         | b         | a | a | a | a | a | a |
| <b>kimenő<br/>kód</b>  | <u>1</u> | <u>2</u> | <u>4</u> | <u>3</u> | <u>5</u> | <u>8</u>  |   | <u>1</u>  | <u>10</u> | <u>11</u> |   |   |   |   |   |   |
|                        | <u>5</u> | <u>7</u> |          |          | <u>9</u> |           |   | <u>11</u> |           |           |   |   |   |   |   |   |
| <b>új string</b>       | <u>4</u> | <u>6</u> |          |          | <u>8</u> | <u>10</u> |   | <u>12</u> |           |           |   |   |   |   |   |   |

- visszaállítás/dekódolás: szótár újra építése
- adatfolyamban!!
- minden kód rekurzívan helyettesítődik a prefix kódjával+ a követő karakterrel
- HW megvalósítás

|                        |          |          |          |   |          |           |           |           |     |
|------------------------|----------|----------|----------|---|----------|-----------|-----------|-----------|-----|
| <b>bemenő<br/>kód</b>  | 1        | 2        | 4        | 3 | 5        | 8         | 1         | 10        | 11  |
|                        | v        | v        | v        | v | v        | v         | v         | v         | v   |
|                        | a        | b        | 1b       | c | 2a       | 5b        | a         | 1a        | 10a |
|                        |          |          | v        |   | v        | v         |           | v         | v   |
|                        |          |          | a        |   | b        | 2a        |           | a         | 1a  |
|                        |          |          |          |   |          | v         |           |           | v   |
|                        |          |          |          |   |          | b         |           |           | a   |
| <b>kimenő<br/>adat</b> | a        | b        | ab       | c | ba       | bab       | a         | aa        | aaa |
| <b>új string</b>       | <u>4</u> | <u>6</u> |          |   | <u>8</u> | <u>10</u> |           | <u>11</u> |     |
|                        |          | <u>5</u> | <u>7</u> |   |          | <u>9</u>  | <u>11</u> |           |     |

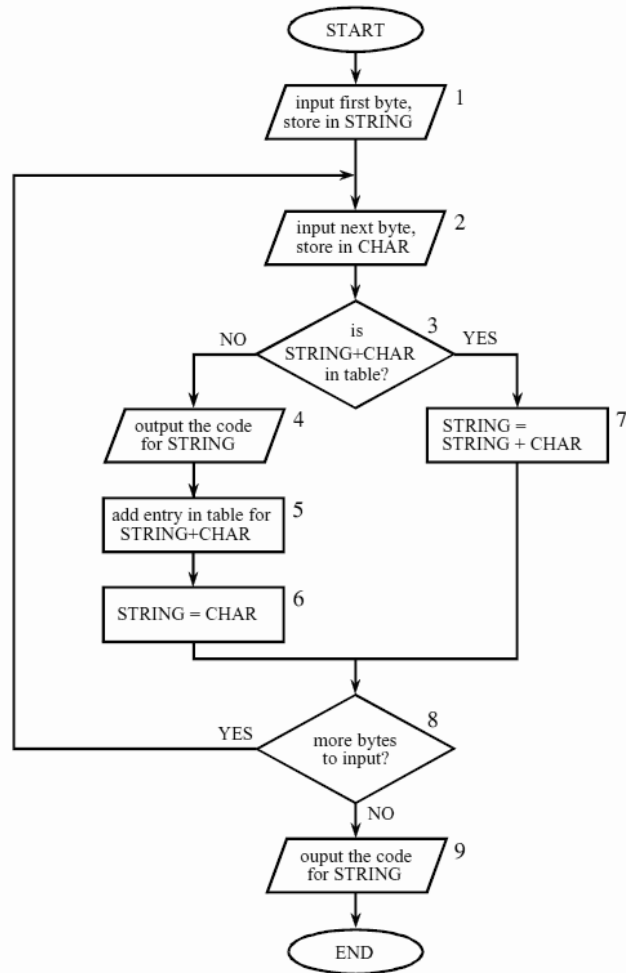


FIGURE 27-7  
LZW compression flowchart. The variable, *CHAR*, is a single byte. The variable, *STRING*, is a variable length sequence of bytes. Data are read from the input file (box 1 & 2) as single bytes, and written to the compressed file (box 4) as 12 bit codes. Table 27-3 shows an example of this algorithm.

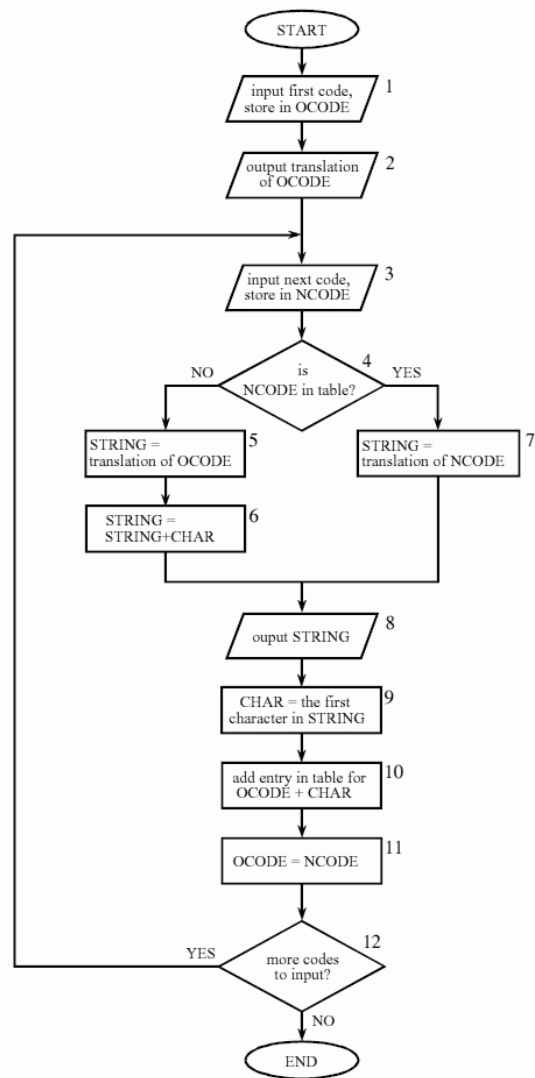
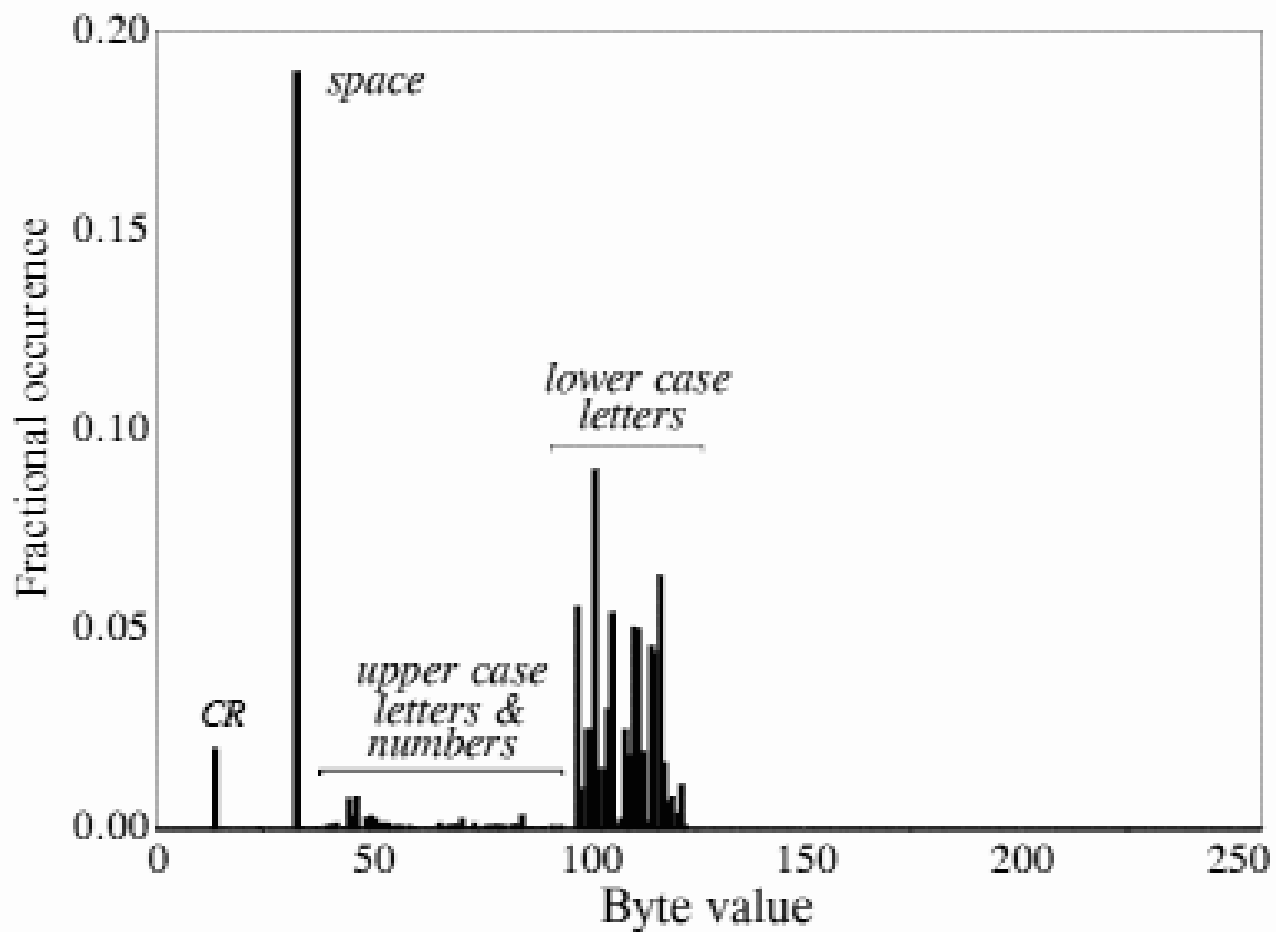


FIGURE 27-8  
LZW uncompression flowchart. The variables, *OCODE* and *NCODE* (oldcode and newcode), hold the 12 bit codes from the compressed file, *CHAR* holds a single byte, *STRING* holds a string of bytes.

Huffman-kódolás:

Bejövő jelek eloszlása előzetesen pontosan ismert



sorrendezés előfordulásuk valószínűségében

gyakorikak - kevés bit

ritkák - sok bit

rekurzívan a két legkisebb valószínűségű jel helyett új jelet vezet be, a két jel

valószínűségének együttes valószínűségével

| Jel   | lépések      |     |     |     |     |
|-------|--------------|-----|-----|-----|-----|
|       | Valószínűség | 1   | 2   | 3   | 4   |
| $a_2$ | 0.4          | 0.4 | 0.4 | 0.4 | 0.6 |
| $a_6$ | 0.3          | 0.3 | 0.3 | 0.3 |     |
| $a_1$ | 0.1          | 0.1 | 0.2 | 0.3 |     |
| $a_4$ | 0.1          | 0.1 | 0.1 |     |     |
| $a_3$ | 0.06         | 0.1 |     |     |     |
| $a_5$ | 0.04         |     |     |     |     |

Kódolás: pl. nagyobb valószínűségű jel 0, a kisebb 1

Visszalépünk egyet a rendezésben, s.í.t.

Prefix kódolás

|       |              | lépések |     |     |     |
|-------|--------------|---------|-----|-----|-----|
| Jel   | Valószínűség | 1       | 2   | 3   | 4   |
| $a_2$ | 0.4          | 0.4     | 0.4 | 0.4 | 0.6 |
| $a_6$ | 0.3          | 0.3     | 0.3 | 0.3 |     |
| $a_1$ | 0.1          | 0.1     | 0.2 | 0.3 |     |
| $a_4$ | 0.1          | 0.1     | 0.1 |     |     |
| $a_3$ | 0.06         | 0.1     |     |     |     |
| $a_5$ | 0.04         |         |     |     |     |

Dekódolás: táblázattal  
Egyértelmű kódhatárok!

|       |      | lépések |          |         |        |       |
|-------|------|---------|----------|---------|--------|-------|
| Jel   | Val. | Kód     | 1        | 2       | 3      | 4     |
| $a_2$ | 0.4  | 1       | 0.4 1    | 0.4 1   | 0.4 1  | 0.6 0 |
| $a_6$ | 0.3  | 00      | 0.3 00   | 0.3 00  | 0.3 00 | 0.4 1 |
| $a_1$ | 0.1  | 011     | 0.1 011  | 0.2 010 | 0.3 01 |       |
| $a_4$ | 0.1  | 0100    | 0.1 0100 | 0.1 011 |        |       |
| $a_3$ | 0.06 | 01010   | 0.1 0101 |         |        |       |
| $a_5$ | 0.04 | 01011   |          |         |        |       |

*Kérdés:* Mekkora a példában szereplő karaktersorozat és annak Huffman kódolásának Shannon-entrópiája?

A statikus kódtábla felépítéséhez ismerni kell a jelet!

Aritmetikai kódolás



## JPEG kódolás

Veszteséges kódolás (l. pl. wavelet) - hosszú kutatómunka, több ajánlás  
 JPEG Baseline coding  
 Kép felosztása  $8 \times 8$ -as területekre (lokálisan adaptív)

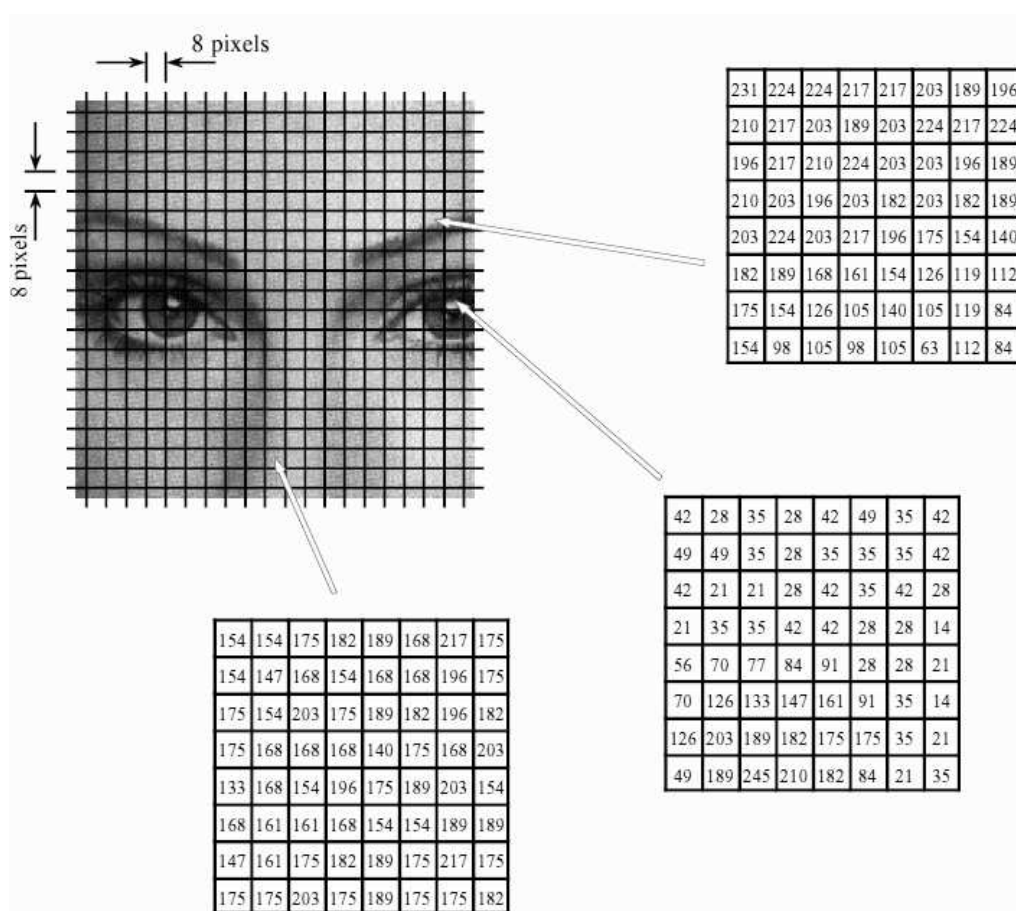
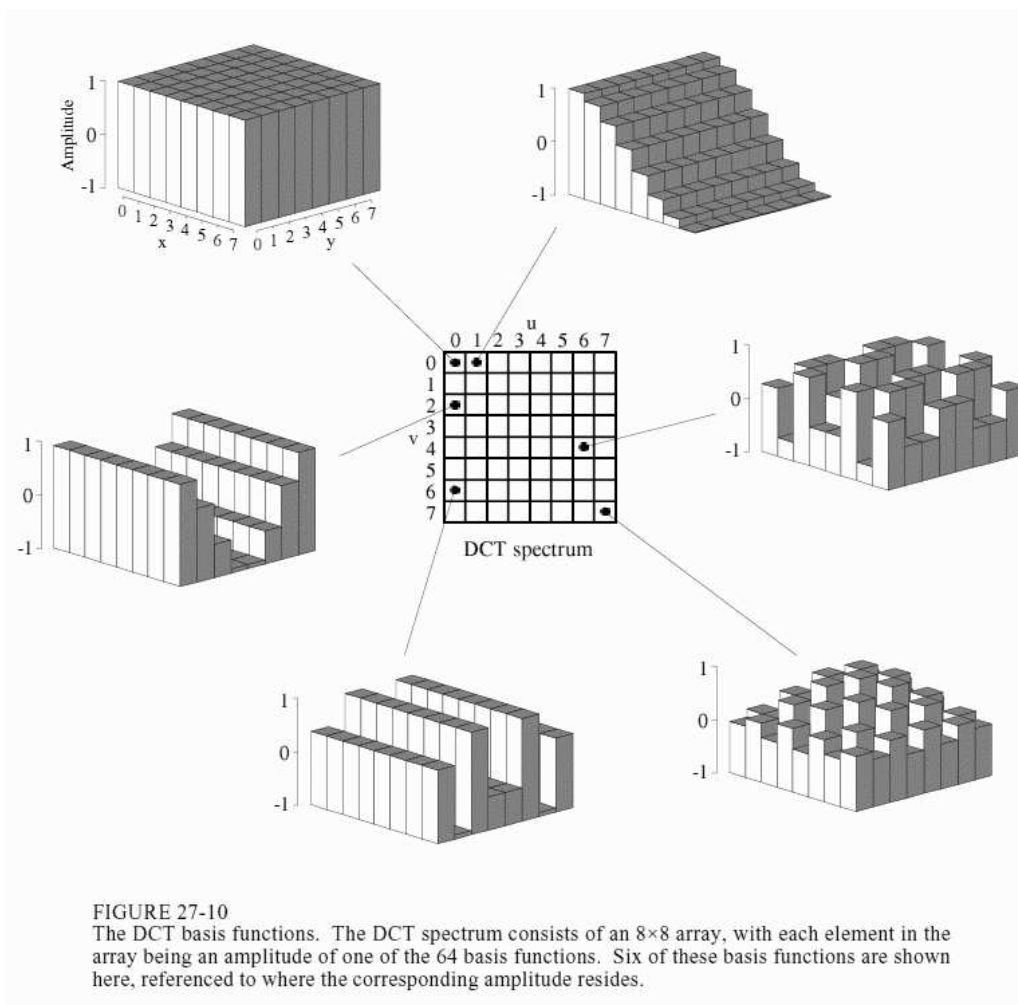


FIGURE 27-9  
 JPEG image division. JPEG transform compression starts by breaking the image into  $8 \times 8$  groups, each containing 64 pixels. Three of these  $8 \times 8$  groups are enlarged in this figure, showing the values of the individual pixels, a single byte value between 0 and 255.

Kódolás blokkonként: Karhunen-Loeve (főkomponens) transzformáció lenne a legjobb, de FFT is jó, és egyszerűbb:

Ne legyen komplex kimenet: diszkrét cosinus transzformációt (DCT) alkalmazunk

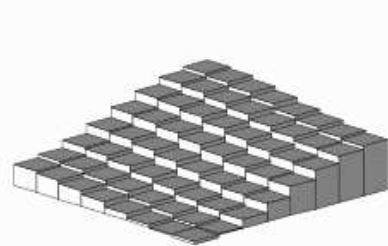
DCT: mintasorrend 1, 2, 3, 4, 5, 6, 7, 8, 7, 6, 5, 4, 3, 2, 1+FFT → szimmetria miatt valós lesz! Azaz  $8 \times 8$  valós értékből  $8 \times 8$  valós értéket készít!



A DCT értékeket kvantáljuk (max. 11 bit!):

| Original Group   | DCT Spectrum | Quantization Error |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
|--|--------------|--------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|-----|-----|-----|----|----|-----|----|---|-----|-----|----|----|----|----|---|----|-----|----|-----|----|----|----|----|----|----|----|----|---|----|----|-----|----|-----|----|----|-----|---|----|----|----|----|----|---|----|----|----|---|---|----|----|----|----|---|---|----|----|---|----|-----|---|----|---|----|---|--|-----|----|----|----|----|---|----|----|-----|---|-----|----|----|---|----|-----|----|-----|---|-----|----|---|----|----|----|----|----|---|---|----|----|----|-----|----|-----|-----|-----|----|----|-----|-----|----|----|----|----|----|----|---|-----|----|----|---|-----|----|---|----|-----|----|----|---|----|----|----|----|
| <p>a. Eyebrow</p> <table border="1"> <tr><td>231</td><td>224</td><td>224</td><td>217</td><td>217</td><td>203</td><td>189</td><td>196</td></tr> <tr><td>210</td><td>217</td><td>203</td><td>189</td><td>203</td><td>224</td><td>217</td><td>224</td></tr> <tr><td>196</td><td>217</td><td>210</td><td>224</td><td>203</td><td>203</td><td>196</td><td>189</td></tr> <tr><td>210</td><td>203</td><td>196</td><td>203</td><td>182</td><td>203</td><td>182</td><td>189</td></tr> <tr><td>203</td><td>224</td><td>203</td><td>217</td><td>196</td><td>175</td><td>154</td><td>140</td></tr> <tr><td>182</td><td>189</td><td>168</td><td>161</td><td>154</td><td>126</td><td>119</td><td>112</td></tr> <tr><td>175</td><td>154</td><td>126</td><td>105</td><td>140</td><td>105</td><td>119</td><td>84</td></tr> <tr><td>154</td><td>98</td><td>105</td><td>98</td><td>105</td><td>63</td><td>112</td><td>84</td></tr> </table>   | 231          | 224                | 224 | 217 | 217 | 203 | 189 | 196 | 210 | 217 | 203 | 189 | 203 | 224 | 217 | 224 | 196 | 217 | 210 | 224 | 203 | 203 | 196 | 189 | 210 | 203 | 196 | 203 | 182 | 203 | 182 | 189 | 203 | 224 | 203 | 217 | 196 | 175 | 154 | 140 | 182 | 189 | 168 | 161 | 154 | 126 | 119 | 112 | 175 | 154 | 126 | 105 | 140 | 105 | 119 | 84  | 154 | 98  | 105 | 98  | 105 | 63  | 112 | 84  | <p>d. Eyebrow spectrum</p> <table border="1"> <tr><td>174</td><td>19</td><td>0</td><td>3</td><td>1</td><td>0</td><td>-3</td><td>1</td></tr> <tr><td>52</td><td>-13</td><td>-3</td><td>-4</td><td>-4</td><td>-4</td><td>5</td><td>-8</td></tr> <tr><td>-18</td><td>-4</td><td>8</td><td>3</td><td>3</td><td>2</td><td>0</td><td>9</td></tr> <tr><td>5</td><td>12</td><td>-4</td><td>0</td><td>0</td><td>-5</td><td>-1</td><td>0</td></tr> <tr><td>1</td><td>2</td><td>-2</td><td>-1</td><td>4</td><td>4</td><td>2</td><td>0</td></tr> <tr><td>-1</td><td>2</td><td>1</td><td>3</td><td>0</td><td>0</td><td>1</td><td>1</td></tr> <tr><td>-2</td><td>5</td><td>-5</td><td>-5</td><td>3</td><td>2</td><td>-1</td><td>-1</td></tr> <tr><td>3</td><td>5</td><td>-7</td><td>0</td><td>0</td><td>0</td><td>-4</td><td>0</td></tr> </table>         | 174 | 19  | 0   | 3  | 1  | 0   | -3 | 1 | 52  | -13 | -3 | -4 | -4 | -4 | 5 | -8 | -18 | -4 | 8   | 3  | 3  | 2  | 0  | 9  | 5  | 12 | -4 | 0 | 0  | -5 | -1  | 0  | 1   | 2  | -2 | -1  | 4 | 4  | 2  | 0  | -1 | 2  | 1 | 3  | 0  | 0  | 1 | 1 | -2 | 5  | -5 | -5 | 3 | 2 | -1 | -1 | 3 | 5  | -7  | 0 | 0  | 0 | -4 | 0 | <p>g. Using 10 bits</p> <table border="1"> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>-1</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>-1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>-1</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>-1</td><td>0</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr> </table>  | 0   | 0  | 0  | 0  | -1 | 0 | 0  | 0  | -1  | 0 | 0   | 0  | 0  | 0 | 0  | -1  | 0  | 0   | 0 | 0   | 0  | 0 | 0  | 0  | 0  | 0  | 0  | 0 | 0 | 0  | 0  | 0  | 0   | 0  | 0   | 0   | 0   | 0  | 0  | 0   | 0   | 0  | 1  | 0  | 0  | 0  | -1 | 0 | 0   | 0  | 0  | 0 | 0   | 0  | 0 | 0  | 0   | 0  | 0  | 0 | 0  | 0  | 0  | 0  |
| 231  | 224          | 224                | 217 | 217 | 203 | 189 | 196 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 210  | 217          | 203                | 189 | 203 | 224 | 217 | 224 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 196  | 217          | 210                | 224 | 203 | 203 | 196 | 189 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 210  | 203          | 196                | 203 | 182 | 203 | 182 | 189 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 203  | 224          | 203                | 217 | 196 | 175 | 154 | 140 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 182  | 189          | 168                | 161 | 154 | 126 | 119 | 112 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 175  | 154          | 126                | 105 | 140 | 105 | 119 | 84  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 154  | 98           | 105                | 98  | 105 | 63  | 112 | 84  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 174  | 19           | 0                  | 3   | 1   | 0   | -3  | 1   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 52   | -13          | -3                 | -4  | -4  | -4  | 5   | -8  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -18  | -4           | 8                  | 3   | 3   | 2   | 0   | 9   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 5  | 12           | -4                 | 0   | 0   | -5  | -1  | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 1  | 2            | -2                 | -1  | 4   | 4   | 2   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -1   | 2            | 1                  | 3   | 0   | 0   | 1   | 1   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -2   | 5            | -5                 | -5  | 3   | 2   | -1  | -1  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 3  | 5            | -7                 | 0   | 0   | 0   | -4  | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | 0            | 0                  | 0   | -1  | 0   | 0   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -1   | 0            | 0                  | 0   | 0   | 0   | 0   | -1  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | 0            | 0                  | 0   | 0   | 0   | 0   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | 0            | 0                  | 0   | 0   | 0   | 0   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | 0            | 0                  | 0   | 0   | 0   | 0   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | 0            | 1                  | 0   | 0   | 0   | -1  | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | 0            | 0                  | 0   | 0   | 0   | 0   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | 0            | 0                  | 0   | 0   | 0   | 0   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| <p>b. Eye</p> <table border="1"> <tr><td>42</td><td>28</td><td>35</td><td>28</td><td>42</td><td>49</td><td>35</td><td>42</td></tr> <tr><td>49</td><td>49</td><td>35</td><td>28</td><td>35</td><td>35</td><td>35</td><td>42</td></tr> <tr><td>42</td><td>21</td><td>21</td><td>28</td><td>42</td><td>35</td><td>42</td><td>28</td></tr> <tr><td>21</td><td>35</td><td>35</td><td>42</td><td>42</td><td>28</td><td>28</td><td>14</td></tr> <tr><td>56</td><td>70</td><td>77</td><td>84</td><td>91</td><td>28</td><td>28</td><td>21</td></tr> <tr><td>70</td><td>126</td><td>133</td><td>147</td><td>161</td><td>91</td><td>35</td><td>14</td></tr> <tr><td>126</td><td>203</td><td>189</td><td>182</td><td>175</td><td>175</td><td>35</td><td>21</td></tr> <tr><td>49</td><td>189</td><td>245</td><td>210</td><td>182</td><td>84</td><td>21</td><td>35</td></tr> </table>  | 42           | 28                 | 35  | 28  | 42  | 49  | 35  | 42  | 49  | 49  | 35  | 28  | 35  | 35  | 35  | 42  | 42  | 21  | 21  | 28  | 42  | 35  | 42  | 28  | 21  | 35  | 35  | 42  | 42  | 28  | 28  | 14  | 56  | 70  | 77  | 84  | 91  | 28  | 28  | 21  | 70  | 126 | 133 | 147 | 161 | 91  | 35  | 14  | 126 | 203 | 189 | 182 | 175 | 175 | 35  | 21  | 49  | 189 | 245 | 210 | 182 | 84  | 21  | 35  | <p>e. Eye spectrum</p> <table border="1"> <tr><td>70</td><td>24</td><td>-28</td><td>-4</td><td>-2</td><td>-10</td><td>-1</td><td>0</td></tr> <tr><td>-53</td><td>-35</td><td>43</td><td>13</td><td>7</td><td>13</td><td>1</td><td>3</td></tr> <tr><td>23</td><td>9</td><td>-10</td><td>-8</td><td>-7</td><td>-6</td><td>5</td><td>-3</td></tr> <tr><td>6</td><td>2</td><td>-2</td><td>8</td><td>2</td><td>-1</td><td>0</td><td>-1</td></tr> <tr><td>-10</td><td>-2</td><td>-1</td><td>-12</td><td>2</td><td>1</td><td>-1</td><td>4</td></tr> <tr><td>3</td><td>0</td><td>0</td><td>11</td><td>-4</td><td>-1</td><td>5</td><td>6</td></tr> <tr><td>-3</td><td>-5</td><td>-5</td><td>-4</td><td>3</td><td>2</td><td>-3</td><td>5</td></tr> <tr><td>3</td><td>0</td><td>4</td><td>5</td><td>1</td><td>2</td><td>1</td><td>0</td></tr> </table> | 70  | 24  | -28 | -4 | -2 | -10 | -1 | 0 | -53 | -35 | 43 | 13 | 7  | 13 | 1 | 3  | 23  | 9  | -10 | -8 | -7 | -6 | 5  | -3 | 6  | 2  | -2 | 8 | 2  | -1 | 0   | -1 | -10 | -2 | -1 | -12 | 2 | 1  | -1 | 4  | 3  | 0  | 0 | 11 | -4 | -1 | 5 | 6 | -3 | -5 | -5 | -4 | 3 | 2 | -3 | 5  | 3 | 0  | 4   | 5 | 1  | 2 | 1  | 0 | <p>h. Using 8 bits</p> <table border="1"> <tr><td>0</td><td>-3</td><td>-1</td><td>-1</td><td>1</td><td>0</td><td>0</td><td>-1</td></tr> <tr><td>1</td><td>0</td><td>-1</td><td>-1</td><td>0</td><td>0</td><td>0</td><td>-1</td></tr> <tr><td>-1</td><td>-2</td><td>1</td><td>0</td><td>-2</td><td>0</td><td>-2</td><td>-2</td></tr> <tr><td>-1</td><td>-2</td><td>-1</td><td>2</td><td>0</td><td>2</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>-2</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>-4</td><td>-1</td><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>-2</td><td>0</td><td>1</td><td>-1</td><td>-1</td><td>1</td><td>-1</td></tr> <tr><td>-1</td><td>-3</td><td>1</td><td>1</td><td>1</td><td>-3</td><td>-2</td><td>-1</td></tr> </table>                                  | 0   | -3 | -1 | -1 | 1  | 0 | 0  | -1 | 1   | 0 | -1  | -1 | 0  | 0 | 0  | -1  | -1 | -2  | 1 | 0   | -2 | 0 | -2 | -2 | -1 | -2 | -1 | 2 | 0 | 2  | 0  | 1  | 0   | -2 | 1   | 0   | 0   | 1  | 0  | 0   | 0   | -4 | -1 | 0  | 1  | 0  | 0  | 0 | 0   | -2 | 0  | 1 | -1  | -1 | 1 | -1 | -1  | -3 | 1  | 1 | 1  | -3 | -2 | -1 |
| 42   | 28           | 35                 | 28  | 42  | 49  | 35  | 42  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 49   | 49           | 35                 | 28  | 35  | 35  | 35  | 42  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 42   | 21           | 21                 | 28  | 42  | 35  | 42  | 28  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 21   | 35           | 35                 | 42  | 42  | 28  | 28  | 14  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 56   | 70           | 77                 | 84  | 91  | 28  | 28  | 21  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 70   | 126          | 133                | 147 | 161 | 91  | 35  | 14  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 126  | 203          | 189                | 182 | 175 | 175 | 35  | 21  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 49   | 189          | 245                | 210 | 182 | 84  | 21  | 35  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 70   | 24           | -28                | -4  | -2  | -10 | -1  | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -53  | -35          | 43                 | 13  | 7   | 13  | 1   | 3   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 23   | 9            | -10                | -8  | -7  | -6  | 5   | -3  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 6  | 2            | -2                 | 8   | 2   | -1  | 0   | -1  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -10  | -2           | -1                 | -12 | 2   | 1   | -1  | 4   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 3  | 0            | 0                  | 11  | -4  | -1  | 5   | 6   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -3   | -5           | -5                 | -4  | 3   | 2   | -3  | 5   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 3  | 0            | 4                  | 5   | 1   | 2   | 1   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | -3           | -1                 | -1  | 1   | 0   | 0   | -1  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 1  | 0            | -1                 | -1  | 0   | 0   | 0   | -1  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -1   | -2           | 1                  | 0   | -2  | 0   | -2  | -2  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -1   | -2           | -1                 | 2   | 0   | 2   | 0   | 1   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | -2           | 1                  | 0   | 0   | 1   | 0   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | -4           | -1                 | 0   | 1   | 0   | 0   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 0  | -2           | 0                  | 1   | -1  | -1  | 1   | -1  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -1   | -3           | 1                  | 1   | 1   | -3  | -2  | -1  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| <p>c. Nose</p> <table border="1"> <tr><td>154</td><td>154</td><td>175</td><td>182</td><td>189</td><td>168</td><td>217</td><td>175</td></tr> <tr><td>154</td><td>147</td><td>168</td><td>154</td><td>168</td><td>168</td><td>196</td><td>175</td></tr> <tr><td>175</td><td>154</td><td>203</td><td>175</td><td>189</td><td>182</td><td>196</td><td>182</td></tr> <tr><td>175</td><td>168</td><td>168</td><td>168</td><td>140</td><td>175</td><td>168</td><td>203</td></tr> <tr><td>133</td><td>168</td><td>154</td><td>196</td><td>175</td><td>189</td><td>203</td><td>154</td></tr> <tr><td>168</td><td>161</td><td>161</td><td>168</td><td>154</td><td>154</td><td>189</td><td>189</td></tr> <tr><td>147</td><td>161</td><td>175</td><td>182</td><td>189</td><td>175</td><td>217</td><td>175</td></tr> <tr><td>175</td><td>175</td><td>203</td><td>175</td><td>189</td><td>175</td><td>175</td><td>182</td></tr> </table> | 154          | 154                | 175 | 182 | 189 | 168 | 217 | 175 | 154 | 147 | 168 | 154 | 168 | 168 | 196 | 175 | 175 | 154 | 203 | 175 | 189 | 182 | 196 | 182 | 175 | 168 | 168 | 168 | 140 | 175 | 168 | 203 | 133 | 168 | 154 | 196 | 175 | 189 | 203 | 154 | 168 | 161 | 161 | 168 | 154 | 154 | 189 | 189 | 147 | 161 | 175 | 182 | 189 | 175 | 217 | 175 | 175 | 175 | 203 | 175 | 189 | 175 | 175 | 182 | <p>f. Nose spectrum</p> <table border="1"> <tr><td>174</td><td>-11</td><td>-2</td><td>-3</td><td>-3</td><td>6</td><td>-3</td><td>4</td></tr> <tr><td>-2</td><td>-3</td><td>1</td><td>2</td><td>0</td><td>3</td><td>1</td><td>2</td></tr> <tr><td>3</td><td>0</td><td>-4</td><td>0</td><td>0</td><td>0</td><td>-1</td><td>9</td></tr> <tr><td>-4</td><td>-6</td><td>-2</td><td>1</td><td>-1</td><td>4</td><td>-10</td><td>-3</td></tr> <tr><td>1</td><td>2</td><td>-2</td><td>0</td><td>0</td><td>-2</td><td>0</td><td>-5</td></tr> <tr><td>3</td><td>-1</td><td>3</td><td>-2</td><td>2</td><td>1</td><td>1</td><td>0</td></tr> <tr><td>3</td><td>5</td><td>2</td><td>-2</td><td>3</td><td>0</td><td>4</td><td>3</td></tr> <tr><td>4</td><td>-3</td><td>-13</td><td>3</td><td>-4</td><td>3</td><td>-5</td><td>3</td></tr> </table>           | 174 | -11 | -2  | -3 | -3 | 6   | -3 | 4 | -2  | -3  | 1  | 2  | 0  | 3  | 1 | 2  | 3   | 0  | -4  | 0  | 0  | 0  | -1 | 9  | -4 | -6 | -2 | 1 | -1 | 4  | -10 | -3 | 1   | 2  | -2 | 0   | 0 | -2 | 0  | -5 | 3  | -1 | 3 | -2 | 2  | 1  | 1 | 0 | 3  | 5  | 2  | -2 | 3 | 0 | 4  | 3  | 4 | -3 | -13 | 3 | -4 | 3 | -5 | 3 | <p>i. Using 5 bits</p> <table border="1"> <tr><td>-13</td><td>-7</td><td>1</td><td>4</td><td>0</td><td>0</td><td>10</td><td>-2</td></tr> <tr><td>-22</td><td>6</td><td>-13</td><td>5</td><td>-5</td><td>2</td><td>-2</td><td>-13</td></tr> <tr><td>-9</td><td>-15</td><td>0</td><td>-17</td><td>-8</td><td>8</td><td>12</td><td>25</td></tr> <tr><td>-9</td><td>16</td><td>1</td><td>9</td><td>1</td><td>-5</td><td>-5</td><td>13</td></tr> <tr><td>-20</td><td>-3</td><td>-13</td><td>-16</td><td>-19</td><td>-1</td><td>-4</td><td>-22</td></tr> <tr><td>-11</td><td>6</td><td>-8</td><td>16</td><td>-9</td><td>-3</td><td>-7</td><td>6</td></tr> <tr><td>-14</td><td>10</td><td>-9</td><td>4</td><td>-15</td><td>3</td><td>3</td><td>-4</td></tr> <tr><td>-13</td><td>19</td><td>12</td><td>9</td><td>18</td><td>5</td><td>-5</td><td>10</td></tr> </table> | -13 | -7 | 1  | 4  | 0  | 0 | 10 | -2 | -22 | 6 | -13 | 5  | -5 | 2 | -2 | -13 | -9 | -15 | 0 | -17 | -8 | 8 | 12 | 25 | -9 | 16 | 1  | 9 | 1 | -5 | -5 | 13 | -20 | -3 | -13 | -16 | -19 | -1 | -4 | -22 | -11 | 6  | -8 | 16 | -9 | -3 | -7 | 6 | -14 | 10 | -9 | 4 | -15 | 3  | 3 | -4 | -13 | 19 | 12 | 9 | 18 | 5  | -5 | 10 |
| 154  | 154          | 175                | 182 | 189 | 168 | 217 | 175 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 154  | 147          | 168                | 154 | 168 | 168 | 196 | 175 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 175  | 154          | 203                | 175 | 189 | 182 | 196 | 182 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 175  | 168          | 168                | 168 | 140 | 175 | 168 | 203 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 133  | 168          | 154                | 196 | 175 | 189 | 203 | 154 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 168  | 161          | 161                | 168 | 154 | 154 | 189 | 189 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 147  | 161          | 175                | 182 | 189 | 175 | 217 | 175 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 175  | 175          | 203                | 175 | 189 | 175 | 175 | 182 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 174  | -11          | -2                 | -3  | -3  | 6   | -3  | 4   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -2   | -3           | 1                  | 2   | 0   | 3   | 1   | 2   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 3  | 0            | -4                 | 0   | 0   | 0   | -1  | 9   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -4   | -6           | -2                 | 1   | -1  | 4   | -10 | -3  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 1  | 2            | -2                 | 0   | 0   | -2  | 0   | -5  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 3  | -1           | 3                  | -2  | 2   | 1   | 1   | 0   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 3  | 5            | 2                  | -2  | 3   | 0   | 4   | 3   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| 4  | -3           | -13                | 3   | -4  | 3   | -5  | 3   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -13  | -7           | 1                  | 4   | 0   | 0   | 10  | -2  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -22  | 6            | -13                | 5   | -5  | 2   | -2  | -13 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -9   | -15          | 0                  | -17 | -8  | 8   | 12  | 25  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -9   | 16           | 1                  | 9   | 1   | -5  | -5  | 13  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -20  | -3           | -13                | -16 | -19 | -1  | -4  | -22 |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -11  | 6            | -8                 | 16  | -9  | -3  | -7  | 6   |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -14  | 10           | -9                 | 4   | -15 | 3   | 3   | -4  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |
| -13  | 19           | 12                 | 9   | 18  | 5   | -5  | 10  |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |     |   |     |     |     |    |    |     |    |   |     |     |    |    |    |    |   |    |     |    |     |    |    |    |    |    |    |    |    |   |    |    |     |    |     |    |    |     |   |    |    |    |    |    |   |    |    |    |   |   |    |    |    |    |   |   |    |    |   |    |     |   |    |   |    |   |  |     |    |    |    |    |   |    |    |     |   |     |    |    |   |    |     |    |     |   |     |    |   |    |    |    |    |    |   |   |    |    |    |     |    |     |     |     |    |    |     |     |    |    |    |    |    |    |   |     |    |    |   |     |    |   |    |     |    |    |   |    |    |    |    |

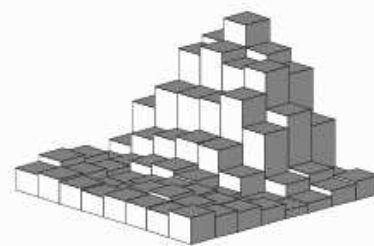
kvantálási tábla:



a. 3 coefficients

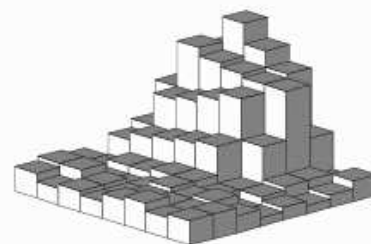


b. 6 coefficients



c. 15 coefficients

FIGURE 27-12  
Example of JPEG reconstruction. The  $8 \times 8$  pixel group used in this example is the *eye* in Fig. 27-9. As shown, less than  $1/4$  of the 64 values are needed to achieve a good approximation to the correct image.



d. 64 coefficients  
(correct image)

a. Low compression

|   |   |   |   |   |   |    |    |
|---|---|---|---|---|---|----|----|
| 1 | 1 | 1 | 1 | 1 | 2 | 2  | 4  |
| 1 | 1 | 1 | 1 | 1 | 2 | 2  | 4  |
| 1 | 1 | 1 | 1 | 2 | 2 | 2  | 4  |
| 1 | 1 | 1 | 1 | 2 | 2 | 4  | 8  |
| 1 | 1 | 2 | 2 | 2 | 2 | 4  | 8  |
| 2 | 2 | 2 | 2 | 2 | 4 | 8  | 8  |
| 2 | 2 | 2 | 4 | 4 | 8 | 8  | 16 |
| 4 | 4 | 4 | 4 | 8 | 8 | 16 | 16 |

b. High compression

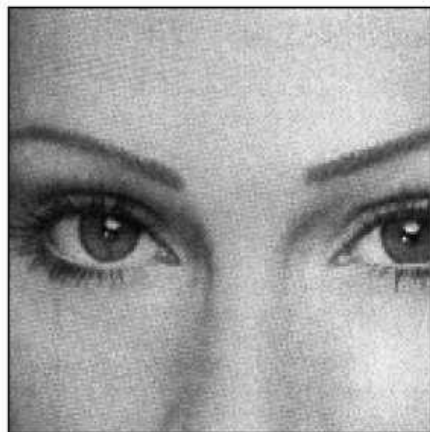
|     |     |     |     |     |     |     |     |
|-----|-----|-----|-----|-----|-----|-----|-----|
| 1   | 2   | 4   | 8   | 16  | 32  | 64  | 128 |
| 2   | 4   | 4   | 8   | 16  | 32  | 64  | 128 |
| 4   | 4   | 8   | 16  | 32  | 64  | 128 | 128 |
| 8   | 8   | 16  | 32  | 64  | 128 | 128 | 256 |
| 16  | 16  | 32  | 64  | 128 | 128 | 256 | 256 |
| 32  | 32  | 64  | 128 | 128 | 256 | 256 | 256 |
| 64  | 64  | 128 | 128 | 256 | 256 | 256 | 256 |
| 128 | 128 | 128 | 256 | 256 | 256 | 256 | 256 |

Lineáris sorozattá alakítás:

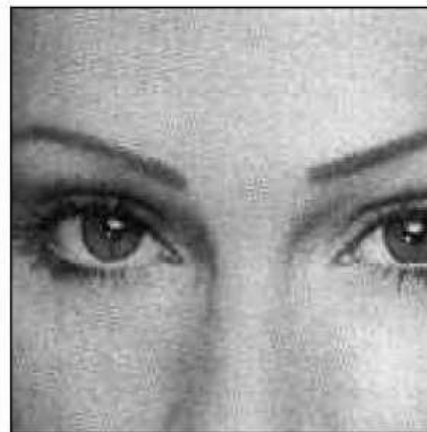
|    |    |    |    |    |    |    |    |
|----|----|----|----|----|----|----|----|
| 0  | 1  | 5  | 6  | 14 | 15 | 27 | 28 |
| 2  | 4  | 7  | 13 | 16 | 26 | 29 | 42 |
| 3  | 8  | 12 | 17 | 25 | 30 | 41 | 43 |
| 9  | 11 | 18 | 24 | 31 | 40 | 44 | 53 |
| 10 | 19 | 23 | 32 | 39 | 45 | 52 | 54 |
| 20 | 22 | 33 | 38 | 46 | 51 | 55 | 60 |
| 21 | 34 | 37 | 47 | 50 | 56 | 59 | 61 |
| 35 | 36 | 48 | 49 | 57 | 58 | 62 | 63 |

egymás mellé kerülnek, a homogén, egyszínű felületek:  
AC komponensek futási hossz kóddal tömörítve

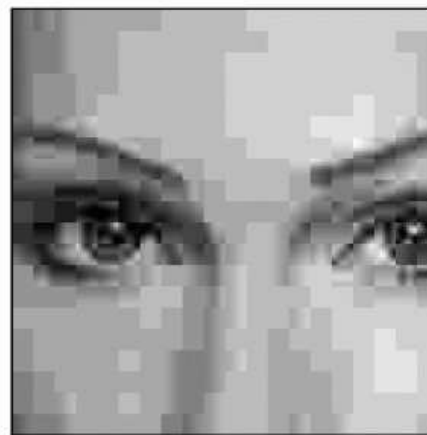
DC komponensek: delta kóddal tömörítve (lassan váltooznak)



a. Original image



b. With 10:1 compression



c. With 45:1 compression

FIGURE 27-15  
Example of JPEG distortion. Figure (a) shows the original image, while (b) and (c) shows restored images using compression ratios of 10:1 and 45:1, respectively. The high compression ratio used in (c) results in each  $8 \times 8$  pixel group being represented by less than 12 bits.

MPEG: mozgó JPEG



## Főkomponens analízis

Mérési adatok  $\rightarrow$  dimenziós vektor

$\phi_i$  ortonormális ( $\phi_i \phi_j = \delta_{ij}$ ) bázis:

$$\mathbf{X} = \sum_{i=1}^n \mathbf{y}_i \phi_i = \Phi \mathbf{Y}$$

itt  $\Phi = [\phi_1 \dots \phi_n]$  és  $\mathbf{Y} = [\mathbf{y}_1 \dots \mathbf{y}_n]^T$ .

Az ortonormalitás miatt  $\mathbf{y}_i = \phi_i \mathbf{X}$

$\mathbf{Y}$   $\mathbf{X}$  elforgatottja

$\phi_i$ : tulajdonság,  $\mathbf{y}_i$  méri

Keressük  $m (< n)$  olyan  $\phi$ -t, amelyik  $\mathbf{X}$ -et legjobban közelíti!

$\mathbf{Y}$  nem használt tagjait (előre meghatározandó)  $b_i$  konstansokkal helyettesítjük:

$$\hat{\mathbf{X}}(m) = \sum_{i=1}^m \mathbf{y}_i \phi_i + \sum_{i=m+1}^n b_i \phi_i$$

Minimalizálás  $\chi^2$  eltérés-négyzet

$$\begin{aligned}
 \chi^2 &= E\|\mathbf{X} - \hat{\mathbf{X}}(m)\| \\
 &= E \sum_{i=m+1}^n \sum_{j=m+1}^n (\mathbf{y}_i - b_i)(\mathbf{y}_j - b_j)\phi_i^T \phi_j \\
 &= \sum_{i=m+1}^n E(\mathbf{y}_i - b_i)^2
 \end{aligned}$$

$E()$ : várható érték operátor

Minimum: deriválni kell  $\chi^2$ -t  $b_i$  szerint!

$$b_i = E\mathbf{y}_i.$$

Ezt visszaírva  $\chi^2$ -be:

$$\begin{aligned}
 \chi^2 &= \sum_{i=m+1}^n E(\mathbf{y}_i - E[\mathbf{y}_i])^2 \\
 &= \sum_{i=m+1}^n \phi_i^T E(\mathbf{X} - E[\mathbf{X}])(\mathbf{X} - E[\mathbf{X}])^T \phi_i \\
 &= \sum_{i=m+1}^n \phi_i^T \Sigma_X \phi_i
 \end{aligned}$$

$\Sigma_X$  az adatok kovariancia mátrixa!

Bebizonyítható, hogy  $\phi_i$ -re az optimum

$$\Sigma_X \phi_i = \lambda_i \phi_i$$

$\phi_i$  a  $\lambda_i$  sajátértékhez tartozó sajátvektor

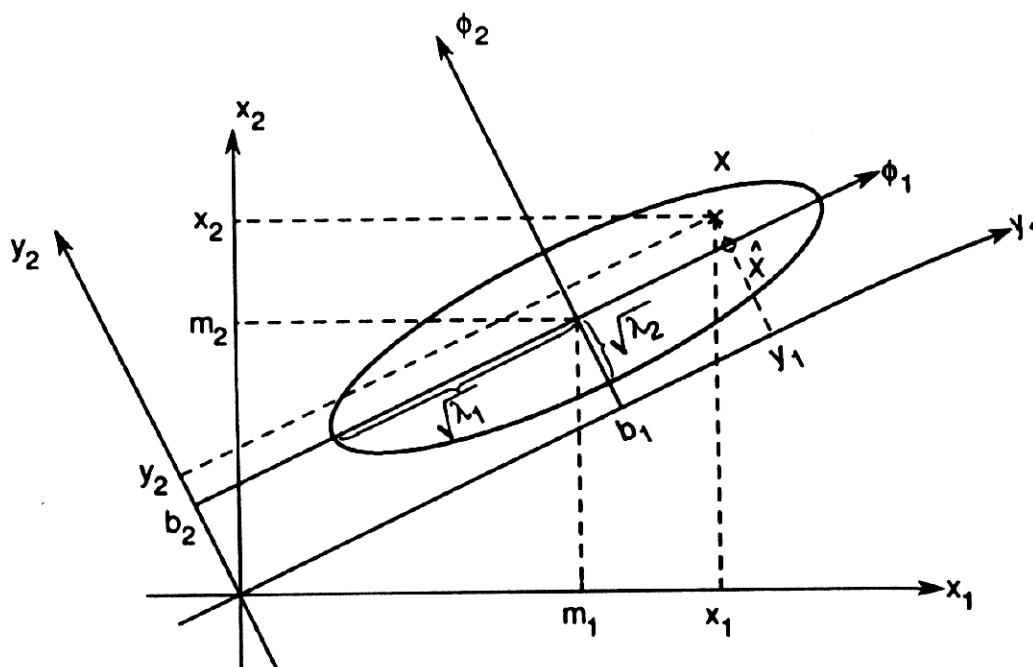
$$\chi^2 = \sum_{i=m+1}^n \lambda_i$$

Azt az eredményt kaptuk tehát, hogy a (a  $\chi^2$  közelítés értelemben) Legjobb lineáris reprezentáció:

kovariancia mátrix sajátvektorai szerinti ortogonális transzformáció alapján!

Ha  $i$  monoton csökken,  $\lambda_i \geq \lambda_j$ , ha  $i > j$

$m (< n)$  „fő” komponens minimalizálja az eltérést!



A  $\phi_1$  és  $\phi_2$  sajátvektorok az eloszlás fő tengelyei

$\lambda_1$  és  $\lambda_2$  sajátértékek:

$\phi_1$  és  $\phi_2$  mentén az eloszlás varianciája

Mivel  $y_i = \phi_i^T \mathbf{X}$ ,

ezért  $y_1$  és  $y_2$  lesz  $\mathbf{X}$  vetületei  $\phi_1$  és  $\phi_2$  tengelyekre.

A  $y_i$  tulajdonságok

ha töröljük az  $y_i$  tulajdonságot, akkor a közelítés hibája  $\lambda_i$ -vel nő meg.

Veszteséges tömörítés:

csak a legnagyobb  $m$  sajátkomponenst és az arra vett vetületek alapján!

Visszaállítás: átlagos eltérés értéke

$$\sum_{i=m+1}^n \lambda_i$$

ha lesz. Ha ez sokkal kisebb, mint

$$\sum_{i=1}^m \lambda_i$$

, és  $m$  sokkal kisebb, mint  $n$ , akkor jelentős tömörítés!

Egyes tulajdonságok egymástól „függetlenek”:

$y_i$  egymás közötti korrelációja 0.

Adatok entrópiájára is szélsőérték:

az összes lineáris transzformáció közül ez a transzformáció minimalizálja a transzformációk  $Y$  terében mért entrópiamaximumot (*minimax* viselkedés)!

Stacionárius idősorok + főkomponens analízis

$y_i$  tulajdonság-függvényei  $e^{j\omega_i t}$  alakúak !  
Visszakapjuk a Fourier-transzformációt!

Főkomponens analízis hátrányai:  
nem mindig (fizikailag) értelmes levonni  $E[\mathbf{X}]$ -t

Csak lineáris tulajdonságok! Ellenpélda: adott síkban körívet leíró adatok

Ezt a hátrányt az adatok Normalizálás:  
 $\mathbf{x}_i$  adatok  $\|\mathbf{x}\|$  szerint normálva  
a főkomponens analízis rendben végrehajtható!  
 $\mathbf{z}_i = \mathbf{x}_i / \sum_{j=1}^n \mathbf{x}_j$  normálás  
 $\Sigma_X$  szinguláris!



## Faktor analízis

Karhunen-Loève (vagy Hotelling) transzformáció.

$$\mathbf{x} = \mathbf{\Lambda} \mathbf{f} + \varepsilon$$

$\mathbf{x}$ : megfigyelés,  $\mathbf{f}$  rejtett változók,  $\varepsilon$  zaj.

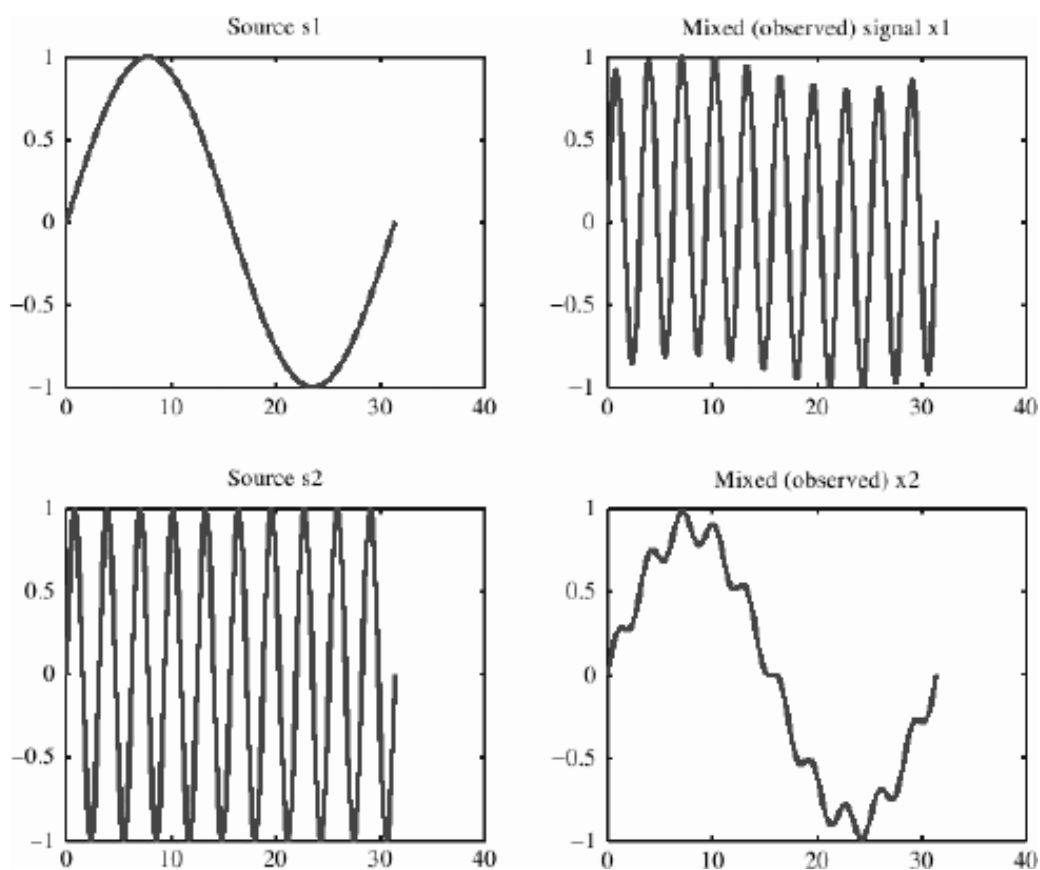
Hasonló a PCA-hoz!

## ICA: Independent Component Analysis

Koktélparti probléma

$$x_1(t) = a_{11}s_1(t) + a_{12}s_2(t)$$

$$x_2(t) = a_{21}s_1(t) + a_{22}s_2(t)$$





Fehérítés + független NEM-GAUSSos komponensek

## Osztályozás

Képek: Zernike/invariáns momentumok.

$$m_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x^p y^q f(x, y) dx dy,$$

ahol  $p, q = 0, 1, 2, \dots$

A centrális momentumok:

$$\mu_{pq} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (x - \bar{x})^p (y - \bar{y})^q f(x, y) dx dy,$$

ahol  $\bar{x} = \frac{m_{10}}{m_{00}}$  és  $\bar{y} = \frac{m_{01}}{m_{00}}$ . Ha  $f(x, y)$  diszkrét függvény:

$$\mu_{pq} = \sum_x \sum_y (x - \bar{x})^p (y - \bar{y})^q f(x, y)$$

Legyen

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{00}^\gamma}, \quad \gamma = \frac{p+q}{2} + 1.$$

$$\phi_1 = \eta_{20} + \eta_{02}$$

$$\phi_2 = (\eta_{20} - \eta_{02})^2 + 4\eta_{11}^2$$

$$\phi_3 = (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2$$

$$\phi_4 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2$$

$$\begin{aligned} \phi_5 = & (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12}) \left[ (\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2 \right] + \\ & + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03}) \left[ 3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2 \right] \end{aligned}$$

$$\phi_6 = (\eta_{20} - \eta_{02}) \left[ (\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2 \right] + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03})$$

$$\begin{aligned} \phi_7 = & (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12}) \left[ (\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2 \right] + \\ & + (3\eta_{12} - \eta_{30})(\eta_{21} + \eta_{03}) \left[ 3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2 \right] \end{aligned}$$

Eltolás, skálázás, forgatás: csoportműveletek,

## Klaszter analízis

| Distance function             | Formula and comments   |
|-------------------------------|--|
| Euclidean distance            | $d(\mathbf{x}, \mathbf{y}) = \sqrt{\sum_{i=1}^n (x_i - y_i)^2}$  |
| Hamming (city block) distance | $d(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^n  x_i - y_i $   |
| Tchebyshev distance           | $d(\mathbf{x}, \mathbf{y}) = \max_{i=1,2,\dots,n}  x_i - y_i $   |
| Minkowski distance            | $d(\mathbf{x}, \mathbf{y}) = \sqrt[p]{\sum_{i=1}^n (x_i - y_i)^p}, p > 0$  |
| Canberra distance             | $d(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^n \frac{ x_i - y_i }{x_i + y_i}, x_i \text{ and } y_i \text{ are positive}$  |
| Angular separation            | $d(\mathbf{x}, \mathbf{y}) = \frac{\sum_{i=1}^n x_i y_i}{\left[ \sum_{i=1}^n x_i^2 \sum_{i=1}^n y_i^2 \right]^{1/2}}$<br>Note: similarity measure expresses the angle between the unit vectors in the direction of $\mathbf{x}$ and $\mathbf{y}$ |

