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A RETROSPECTION OF CHAOTIC PHENOMENA IN ELECTRICAL SYSTEMS

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In the last decade new phenomena have been observed in all areas of non linear dynamics, principal among these being 'Chaotic phenomena'. Chaos has been reported virtually from every scientific discipline. This paper summarizes a study of the chaotic phenomena in electrical systems and attempts to translate the mathematical ideas and techniques into language that engineers and applied scientists can use to study 'Chaos'. Towards this end, the paper has summarized the study of chaos in several examples like Chua's circuit family; Folded Torus circuit; non-autonomous circuits; switched capacitor circuits and hyper-chaos circuits. As observed in power systems, control systems and digital filters, chaos has been exhibited and shown on examples.

Keywords: Choas; phenomena; nonlinear dynamics; nonlinear systems

HISTORY AND BACKGROUND OF CHAOS

The nonscientific concept of 'Chaos' is very old and often associated with a physical state or human behaviour without pattern and out of control. Many scientists observed it long ago, but dismissed it only as a physical noise. The term 'Chaos' often stirs fear implying that governing laws or traditions no longer have control over events such as prison riots, civil war, or a world war. There is always some underlying force or reason behind the chaos that can explain why seemingly random events appear unpredictable. In the physical sciences, the paragon of chaotic phenomena is turbulence. Thus, a rising column of smoke or the eddies behind a boat or aircraft wing provide graphic examples of chaotic motion. Chaotic dynamics are thus inherent in all of nonlinear physical phenomena that has created a sense of revolution in physics today.

We must distinguish between the so called random and chaotic motions. In random motions, we truly do not know the input forces or we only know some statistical measures of the parameters. The term 'Chaotic' is reserved for deterministic problems. Henri Poincare [1] showed the existence of 'Chaotic' or unpredictable motions from the classical equations of physics. "Chaotic" is thus a term assigned to that class of motions in deterministic physical and mathematical systems whose time history has a sensitive dependence on initial conditions. Thus unless a computer of infinite word length is used in the simulation, no long-term prediction of the precise solution waveform is possible.

An excellent discussion of uncertainties and determinism and Poincare's ideas may be found in the very readable book by L. Brillouin [2]. The growth in uncertainty is given by $N = N_0 e^{ht}$ which is another property of chaotic systems. The constant h is related to the concept of 'entropy' in information theory and also to another concept called the 'Lyapunov exponent'.

Lyapunov exponents are used to measure local expansions and contractions near an attractor of a dynamical system. The Lyapunov exponent if positive shows the existence of chaos. There is currently some work on 'averaged' Lyapunov exponents to detect transitions to spatio temporal chaos, with application to fluids, electronic oscillators *etc.*

The subject of 'Chaos' has become newsworthy – particularly the study of mathematical chaos. Many popular science magazines and even 'The New York Times' and 'Newsweek' have carried articles on the new studies into mathematics of chaotic dynamics. Engineers have always known about chaos. For example, Birkhoff [3] was aware of chaotic solutions since the turn of the century; Van der Pol and Van der Mark [4] reported 'irregular noise' in experiments with an electronic oscillator in the magazine 'Nature'.

The new thing about 'Chaotic Dynamics' is the discovery of a seemingly underlying order which holds out the promise of being able to predict certain properties of this 'noisy behaviour'. Chaotic vibrations occur whenever some strong nonlinearity exists. Examples of nonlinearities is electrical systems include the following or capacitive circuit element; Diodes; many transistors and other active devices; Electric and magnetic forces; nonlinear feedback control forces *etc*.

Chaotic phenomena have been observed in many physical systems. Each month new phenomena are reported in the scientific and engineering literature. A few of the many phenomena in which chaos has been uncovered included the following: Nonlinear acoustic systems; simple forced circuits with diodes: Harmonically forced circuits with $p-n$ transistor elements; Harmonically forced circuits with nonlinear capacitance and inductance elements and feedback control devices. 'Chaos' is therefore so pervasive and specific manifestation of 'Chaotic' solutions have arrived with the arrival of powerful computers.

THEORY OF CHAOS

Ueda of the Kyoto University in Japan was one of the first ones to discover the chaos in a nonlinear inductor electrical circuit [5]. If we choose the 'cubic' current controlled $i-v$ characteristic, the resulting circuit is called the Van der Pol oscillator in the state equations as:

$$\dot{V}_c = \frac{1}{\sqrt{LC}} \frac{d\,Vc}{d\tau}$$

and

$$iL = \frac{1}{\sqrt{LC}} \frac{diL}{d\tau},$$

where

$$\tau \triangleq \frac{1}{\sqrt{LC}} \tau.$$

If we define $x_1 \triangleq iL$ and $x_2 = d\,iL/d\tau$, the state equation can be recast as:

$$\frac{dx_1}{d\tau} = x_2$$

and

$$\frac{dx_2}{d\tau} = \varepsilon(1 - x_1^2)\, x_2 - x,$$

where

$$\varepsilon \triangleq \sqrt{\frac{C}{L}}$$

These equations can be further recast into the following equivalent scaler nonlinear second-order differential equation changing x_1 to x as

$$\ddot{x} + \varepsilon(x^2 - 1)\,\dot{x} + x = 0.$$

Similarly we can derive Duffing's equations also.

Both the Duffing's and Van der Pol equations have been studied for decades. There are examples of other nonlinear circuits also exhibiting chaos [6].

CHAOTIC BEHAVIOUR IN ELECTRONIC CIRCUITS

Example 1

Chua's Circuit Famxy

Chua's circuit (Fig. 1) is one of the simplest autonomous circuit which can become chaotic. It is a text book example of chaos and contains only three energy storage elements (The minimum number needed for a dynamic system to be chaotic) and only one nonlinear element of the simplest type, *viz.* a two-terminal piecewise-linear resistor. It is the only known example of a physical system which has been shown to be chaotic using three different approaches: computer simulation, laboratory experiments, and mathematical analysis, Shuxian Wu [7] has generalized Chua's circuit into a 6-parameter family of potentially chaotic circuits – The Chua's circuit family.

Chua's circuit exhibits an immense variety of non-linear dynamical phenomena, including many typical 'bifurcation' and 'routes to chaos'. In fact, it is a prototype model of chaos which provides a

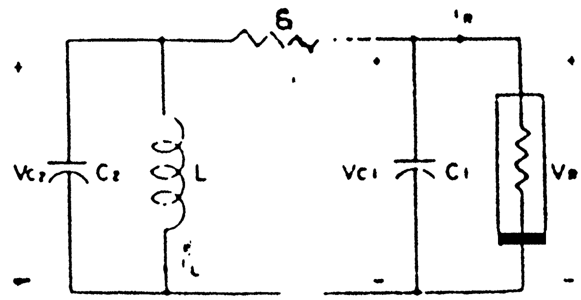

FIGURE 1

quick and broad introduction to the subject of chaos. Matsumoto, Chua and Komuro [8] presented the response of Chua's circuit:- The double scroll (Fig. 2). A simple two-transistor realization of this circuit [9] is depicted in Figure 3. The two transistors of Figure 3 can be replaced by an op-amp as shown in Figure 4 which is still simpler and further requires lesser battery voltage.

Example 2

Folded Torus Circuit

This is another very simple electronic circuit suggested in Langford's Book [10] and is shown in Figure 5(a) along with its physical practical realization in Figure 5(b). It is a simple third-order autonomous circuit consisting of only four elements – only one of which is a nonlinear resistor. With the use of two different values of C_1, we get a '2-torus' and a 'folded torus' chaotic attractor as displayed in Figures 6(a) and (b) respectively.

NONAUTONOMOUS CHAOTIC CIRCUITS

Dynamic circuits containing only linear time-invariant-elements and dc sources are called autonomous circuits.

Chua *et al.*, have presented a simplest nonautonomous circuit also [11] and [12] gives a Josephson – junction circuit in which Melnikov's method can be used to prove the presence of chaotic behaviour.

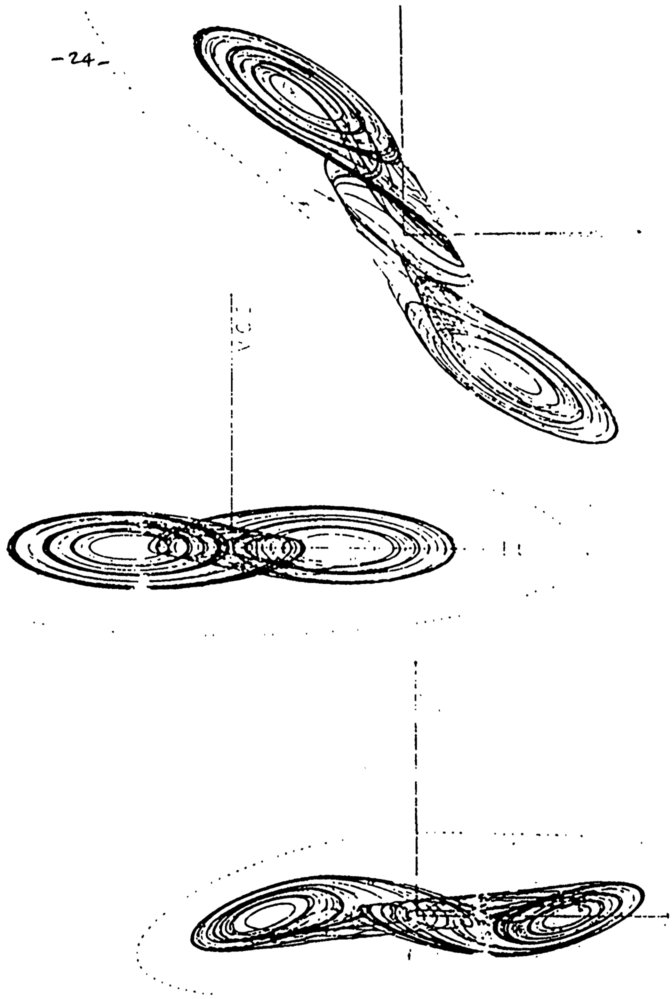

FIGURE 2

SWITCHED CAPACITOR CHAOTIC CIRCUITS

Huertas *et al.* [13] have proposed a switched capacitor circuit with chaotic behaviour and also a switched-capacitor analog computer [14] for simulating chaotic and bifurcation phenomena.

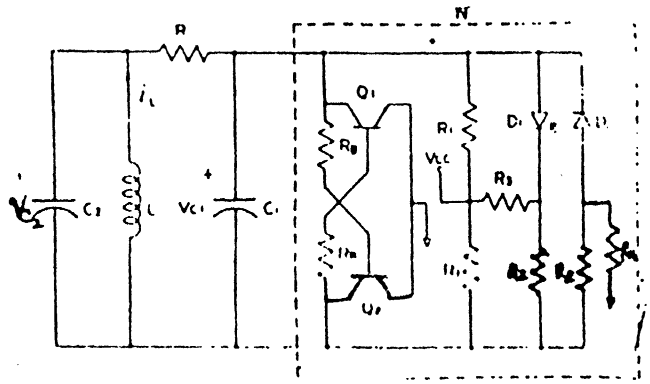

FIGURE 3

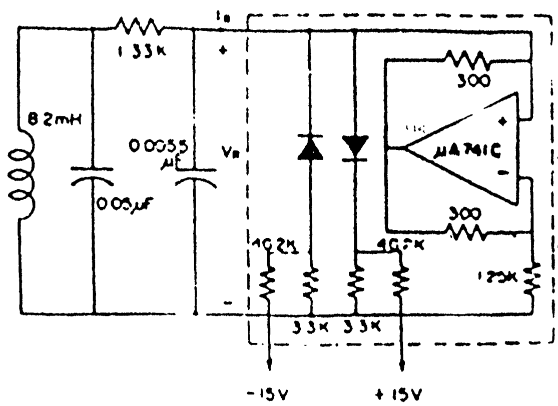

FIGURE 4

HYPER-CHAOS CIRCUITS

Chua *et al.* [15] have given yet another interesting circuit which exhibits a chaotic attractor with more than one positive Lyapunov exponents. The dynamics expands not only small line segments but also small area elements thereby giving rise to a 'thick' attractor. This is the first real physical system where a phyerchaos has been observed experimentally and confirmed by computer [16] . The practical circuit is illustrated in Figure 7 along with the digitally simulated attractors in Figure 8.

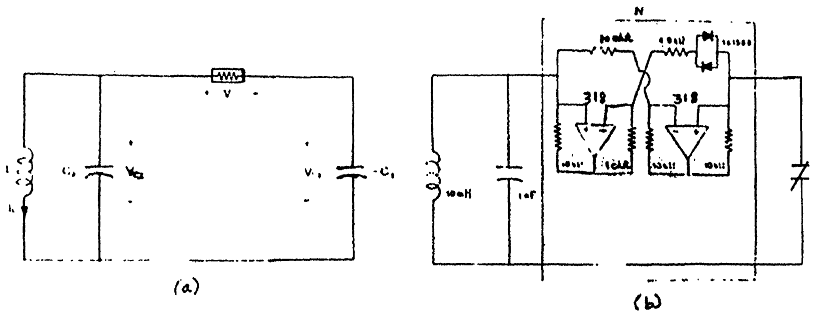

FIGURE 5

CHAOS OBSERVED IN POWER SYSTEMS

Chen and P. P. Varaiya [17] have reported the occurence of degenerate Hopf bifurcation in power systems. The Hopf bifurcations refer to the development of periodic orbits ("self oscillations") from a stable fixed point) as a parameter a critical value. This power system is shown in Figure 9 as an example.

Varaiya, Wu and Chen [18]; Salam, Marsden and Varaiya [19, 20] have also exhibited chaotic motion (called Arnold diffusion) in power system.

CHAOS IN COMNOL SYSTEMS

Two types of chaotic vibrations problems can be explored under this. It depends on the reference signal $X_r(t)$. Which could either be zero when the system is autonomous or could be periodic when the motion is over a given path and over again as in some manufacturing robotic device.

Holmes [21] has studied the chaotic vibrations for an autonomous system which has been shown to exhibit both periodic limit cycle oscillations as well as chaotic motion. The problem of a forced feedback system has been studied by Golnaraghi and Moon [22] from Cornell University, Ithaca, NT, USA.

Chaos generation has been domonstrated also in adaptive control systems in two articles [23] and [24] in the IEEE Trans. Circuits and

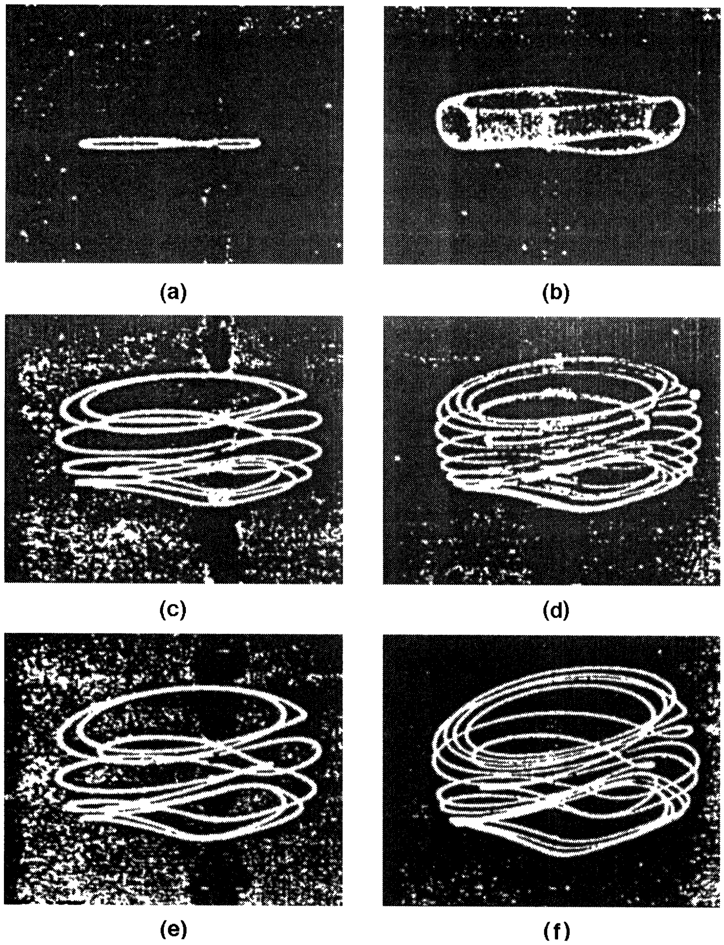

FIGURE 6a

Systems: Special Issue on Chaos and Bifurcations and Circuits and Systems, July, 1980.

CHAOS IN DIGITAL FILTER

In June, 1988, Chua and Lin [25] showed that the second-order digital filter, when implemented using a 2's complement arithmetic for

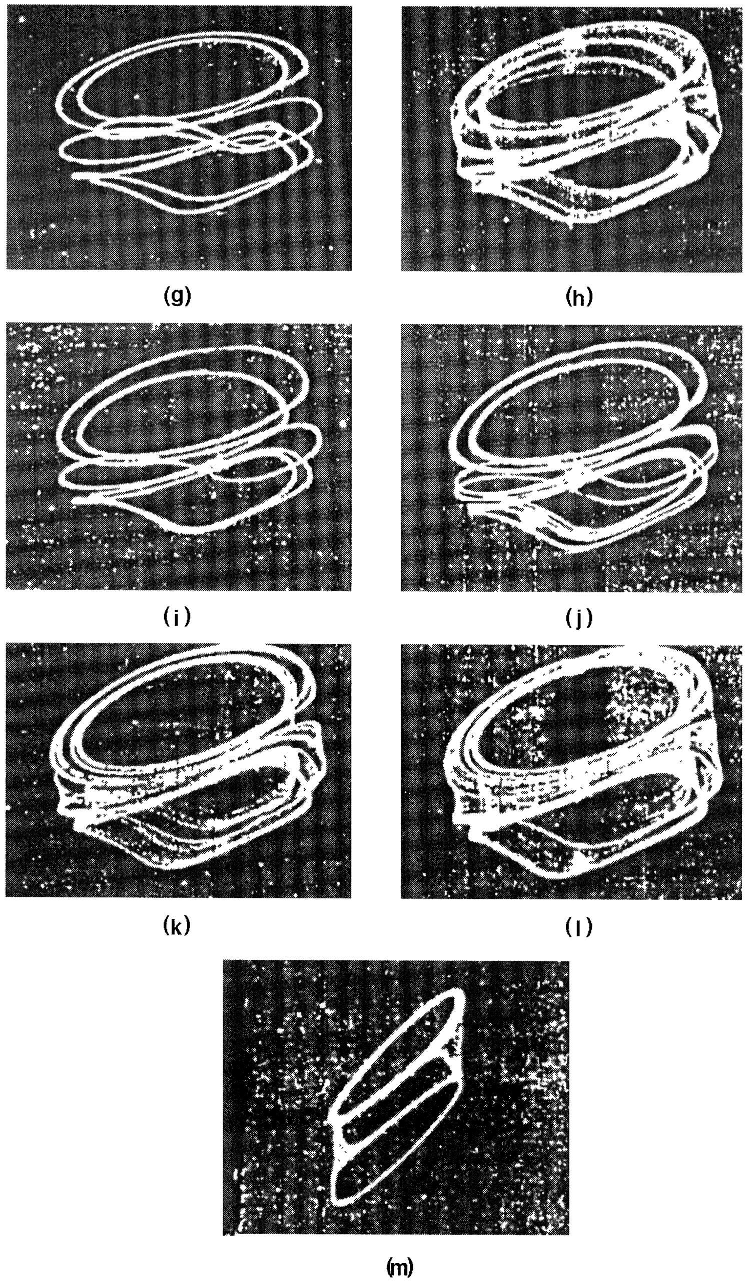

FIGURE 6b

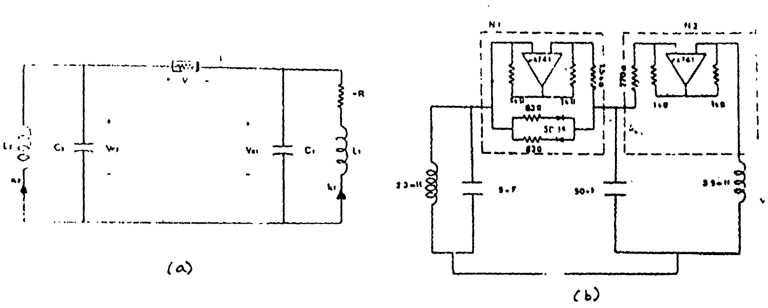

FIGURE 7

addition operation, can exhibit chaotic behaviour for certain region in the parameter space. They extended the study to third-order digital filters also [26] observing much richer dynamics with an overflow nonlinearity. Computer simulations show that incredibly complicated geometrical patterns are exhibited.

The above two publications aroused great interest in the digital signal processing community and some hardware implementation AT and T Bell Lab [27] and software packages were developed Prof. Tony Davies whom at City University, London [28, 29], Lin and Chua again have proved in a very recent letter [30] that the chaotic nature of a real digital filter may be hidden because of short word lengths, but the chaotic behaviour must be considered in a real digital filter when the word length exceeds.

EDUCATIONAL APPLICATIONS OF CHAOS

A. I. Mees and Colin Sparrow in the Proc. IEEE Special Issue, August, 1987 [31] have described the two theoretical techniques, *viz.* The Melnikov and the Shilnikov Bifurcation Theorems which will provide full understanding of the complex dynamical chaotic behaviour in nonlinear systems. Also in the same issue Parker and Chua [32] have discussed a software Tool Kit named INSITE for the analysis of nonlinear dynamical systems. The tool kit runs under both PC-DOS and the UNIX operating systems and calculates and displays trajectories, bifurcation diagrams and two-dimensional phase portraits.

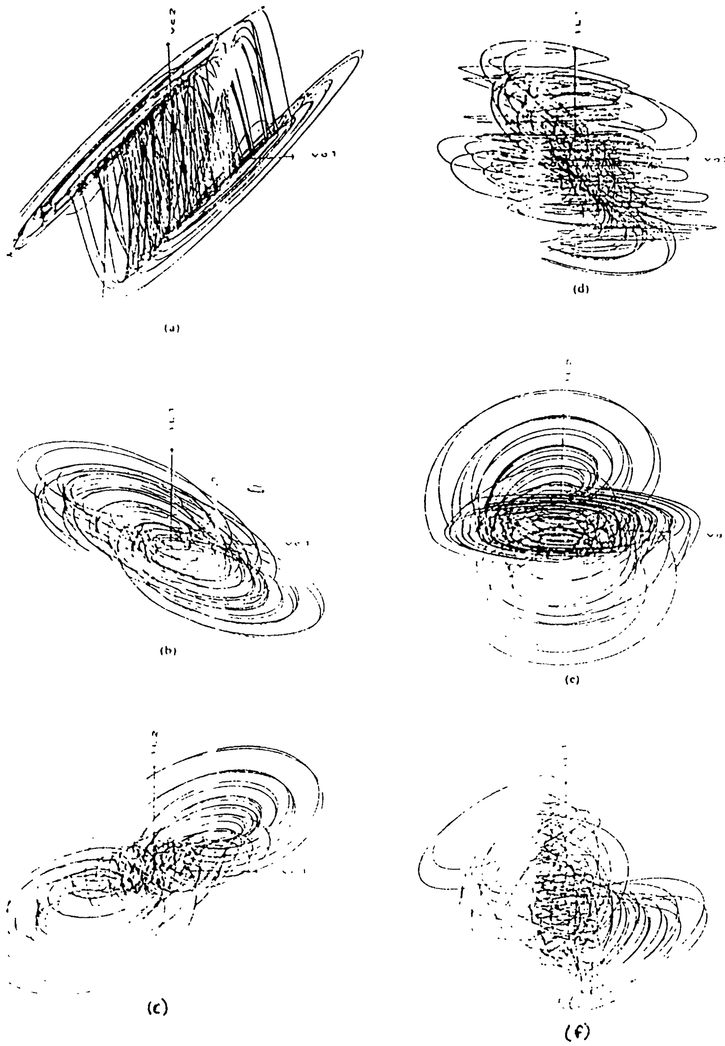

FIGURE 8

Chua and Sugawara yet again in another article [33] in the same special issue have developed an electronic instrument (analog and not digital) which is small and portable and when used as a preprocessor with a standard oscilloscope, allows one to display the strange attractors observed from chaotic circuits.

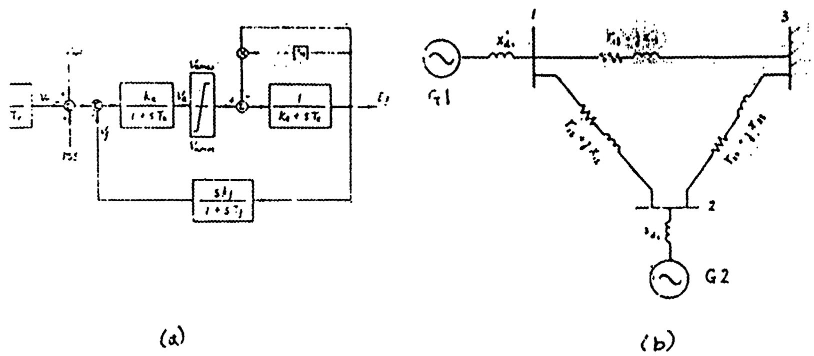

FIGURE 9

CONCLUSIONS

'Chaos', also called stange behaviour, is currently one of the most exciting topics in nonlinear systems research. The filed of chaotic dynamics is rapidly growing. To summarize: The most spectacular manifestation of strong nonlinearity is the completely irregular behaviour of very simple circuits whose nonlinear elements have perfectly regular characteristics. This is generally called 'Chaotic Behaviour' Chaos is ubiquitous; all pervasive and permeates all the fields of science universally.

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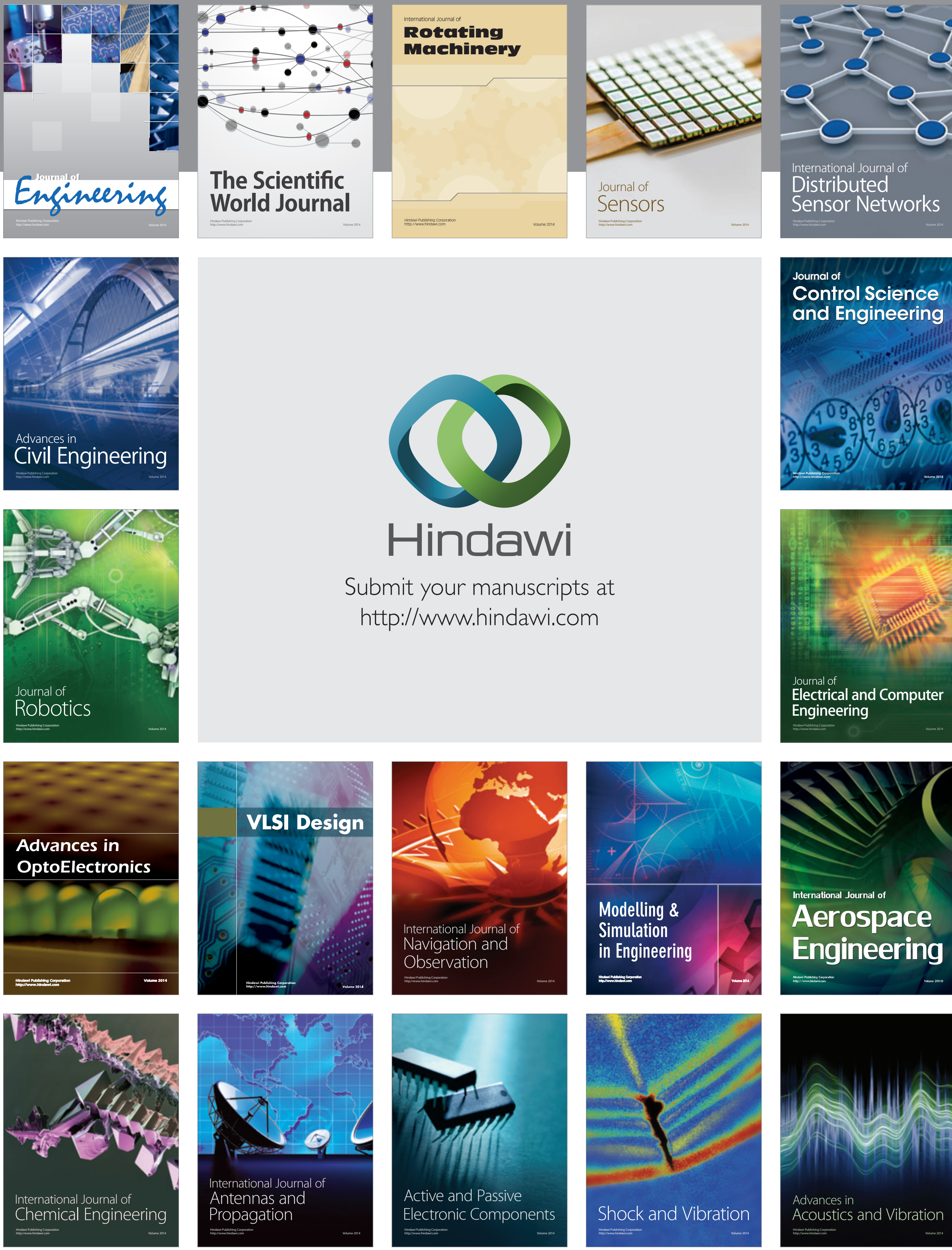
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