$n-$Scroll chaos generators: a simple circuit model

M. E. Yalçın, S. Özoğuz, J.A.K. Suykens and J. Vandewalle

In this letter, we present n-scroll attractors which are generated from a generalized and simple circuit model. The nonlinear characteristics can be systematically designed by adding comparators. We show a 5-scroll attractor circuit realization which has been experimentally verified by using current feedback opamps.

Introduction: The use of chaotic signals in communications has recently received a great deal of interest. An important part in chaos-based analog/digital communications systems [1] is the choice of the chaotic oscillator. Chua's circuit [2] is probably the most well-known and commonly used chaotic oscillator in this field. Among the many generalizations of Chua's circuit, more complicated attractors have been proposed by Suykens & Vandewalle [3] by introducing additional breakpoints in the nonlinearity of Chua's circuit, leading to so-called n-double scroll attractors. A more complete family of n-scroll instead of n-double scroll attractors has been obtained from a generalized Chua's circuit reported in [4]. Experimental confirmations of 2-double scroll and 5-scroll attractors have been given in [5] and [6], respectively. The same generalization idea has been applied to n-scroll hyperchaotic attractors proposed by Yalcin, Suykens & Vandewalle [7]. The design of chaos generators has received considerable attention and as a result a novel and simple model for chaos generation has been recently proposed in [8]. A double scroll-like attractor has been observed from this circuit. Here, we show how the simple model proposed in [8] can be generalized in order to generate n-scroll attractors, in a somewhat similar fashion as the original Chua's circuit has been generalized in order to obtain such a family of attractors.

As a result of the well-known advantages of the current feedback opamps (CFOAs) over the conventional opamps, i.e. much higher slew rates and constant bandwidth almost independent of the gain, researchers have attempted to use CFOAs in the implementation of chaotic oscillators in order to have an improved high-frequency performance, e.g. [9, 10]. According to this, as a possible implementation of the proposed generalized model, we present a circuit, whose core is implemented using CFOAs.

Chaos generator with hard limiter and its generalization: In [8] the following chaos generator with hard limiter has been proposed

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -a & -a \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ a \end{bmatrix} f(x_1), \quad f(x_1) = \begin{cases} 1 & x_1 \geq 0 \\ -1 & x_1 < 0 \end{cases} \tag{1}$$

where $x \in \mathbb{R}^3$. It has been observed that the proposed model Eq.(1) exhibits a double-scroll-like attractor by setting $a = 0.8$. In this letter, the main idea is to modify the nonlinear characteristic $f(x_1)$ into

$$f(x_1) = \sum_{i=1}^{N} a_i f_{p_i}(x_1) + \sum_{j=1}^{M} b_j \left(f_{n_j}(x_1) - 1 \right) \tag{2}$$

where

$$f_m(x_1) = \begin{cases} 1 & x_1 \geq m \\ 0 & x_1 < m \end{cases} \tag{3}$$

and N, M denote natural numbers, $a_i = \begin{cases} 1 & i=1 \\ 2 & i>1 \end{cases}$, $b_j = \begin{cases} 1 & j=1 \\ 2 & j>1 \end{cases}$, $p_i = 2(i-1)$, $n_j = -2(j-1)$. In [8], the special case of $N=1, M=1$ has been investigated for this generalized characteristic. In the generalized nonlinearity, the core function $f_m(\cdot)$ is a comparator which is a useful aspect for circuit realization. The set of equilibrium points of the generalized system is given by $\{-2M+1, ..., -2j+1, ..., -1, 1, ...2i-1, ...2N-1\}$. The Jacobian matrices evaluated at each of these equilibrium points are the same. The scrolls are located around these equilibrium positions and the number of scrolls generated from the generalized nonlinearity is equal to $N+M$. A computer simulation for the 8-scroll attractor is shown in Fig. 1 corresponding to $N=4$, $M=4$, $a=0.4$.

Circuit realization: The circuit realization of the proposed chaotic oscillator is given in Fig. 2. The core of the circuit is implemented by means of CFOAs. Hence, the chaotic oscillator is expected to offer an improved high-frequency performance. For $R_1 = R_2 = aR_3 = R$, $C_1 = C_2 = C_3 = C$ and using the normalized quantities $t_n = t/RC$, $x_1 = I_{R1}$, $x_2 = I_{C1}$, $x_3 = I_{C3}$, $a_i = V_{CC}/R_{ai}$, $(i = 1,2)$, $p_2 = V_{p2}/R$, $b_j = V_{CC}/R_{bj}$, $n_j = V_{nj}/R$, $(j = 2,3)$ $b_1 = a_1$, it can be verified that the circuit realizes Eqs.(1) and (2) for $M=3$ and $N=2$. Using this circuit, the 5-scroll attractor can be observed. It should be noted that systematic generation of a higher number of scrolls is possible by using additional comparators.

The circuit in Fig. 2 is verified experimentally using the commercial CFOA, AD844 [11] supplied under $\pm 10\,V$ and LM311 type comparators. The passive component values have been chosen as $R_1 = R_2 = 1k\Omega$, $R_3 = 1.9k\Omega$ corresponding to $a \simeq 0.53$, $R_{a1} = 2R_{b2} = 2R_{b3} = 2R_{a2} = 15k\Omega$, $C_1 = C_2 =$

$C_3 = 1nF$ and also V_{CC} is taken as $10\,V$. The controlling voltages at the comparators noninverting inputs, i.e. V_ns and V_ps are taken as adjustable. The observed (V_{R1}, V_{R3}) trajectory corresponding to the $(x_1, (-x_3))$ trajectory from which a 5-scroll attractor can be observed is given in Fig. 3.

Conclusion: We have introduced a generalization of a simple circuit model for double-scroll like chaos generation, which results into more complicated n-scroll attractors. A circuit realization has been made by using CFOAs, which is expected to offer a good high-frequency performance.

Acknowledgments: This research work was carried out at the ESAT laboratory and the Interdisciplinary Center of Neural Networks ICNN of the Katholieke Universiteit Leuven, in the framework of the the Belgian Programme on Interuniversity Poles of Attraction, initiated by the Belgian State, Prime Minister's Office for Science, Technology and Culture (IUAP P4-02), the Concerted Action Project MEFISTO of the Flemish Community and ESPRIT IV 27077 (DICTAM). Johan Suykens is a postdoctoral researcher with the National Fund for Scientific Research FWO - Flanders.

References

1 KOLUMBAN, G., KENNEDY, M.P. and CHUA, L.O.: 'The role of synchronization in digital communications using chaos. I . Fundamentals of digital communications', IEEE Trans. Circuits and Systems-I, 1997, 44, (10), pp. 927-935.

2 CHUA, L.O., KOMURO, M. and MATSUMOTO, T.: 'The Double Scroll Family', IEEE Trans. Circuits and Systems-I, 1986, 33, (11), pp. 1072-1118.

3 SUYKENS, J.A.K. and VANDEWALLE, J.: 'Generation of n-double scrolls ($n= 1,2,3,4,...$)', IEEE Trans. Circuits and Systems-I , 1993, 40, (11), pp. 861-867.

4 SUYKENS, J. A. K., HUANG, A. and CHUA, L. O.: 'A Family of n-Scroll Attractors from a Generalized Chua's Circuit', AEÜ Int. J. Electron. Commun, 1997, 51, (3), pp. 131-138.

5 ARENA, P., BAGLIO S., FORTUNA, L. and MANGANARO, G.: 'Generation of n-double scrolls via cellular neural networks', Int. J. Circuit Theory and Applications, 1996, 24, 241-252.

6 YALCIN, M. E., SUYKENS, J.A.K. and VANDEWALLE, J.: 'Experimental confirmation of 3- and 5-scroll Attractors from a Generalized Chua's Circuit', IEEE Trans. Circuits and Systems-I, 2000, 47, (3), pp. 425-429.

7 YALCIN, M. E., SUYKENS, J.A.K. and VANDEWALLE, J.: 'Hyperchaotic n-scroll attractors', Proceedings of the IEEE Workshop on Nonlinear Dynamics of Electronic Systems (NDES 2000), 2000, pp. 25-28.

8 ELWAKIL, A. S., SALAMA, K. N. and KENNEDY, M. P.: 'A System for Chaos Generation and Its Implementation in Monolithic Form', Proceedings of IEEE Int. Symp. on Circuits and Systems (ISCAS 2000), 2000, (V), pp. 217-220.

9 SENANI, R., GUPTA, S.S.: 'Implementation of Chua's chaotic circuits us-

ing current feedback opamps', Electron. Lett., 1998, 34, (9), pp. 829-830.

10 ELWAKIL, A. S. and KENNEDY, M. P.: 'Improved implementation of Chua's chaotic oscillator using current feedback op amp', IEEE Trans. Circuits and Systems-I, 2000, 47, (1), pp. 76-79.

11 ANALOG DEVICES, Linear products data book, (Norwood, Massachusetts, USA, 1990).

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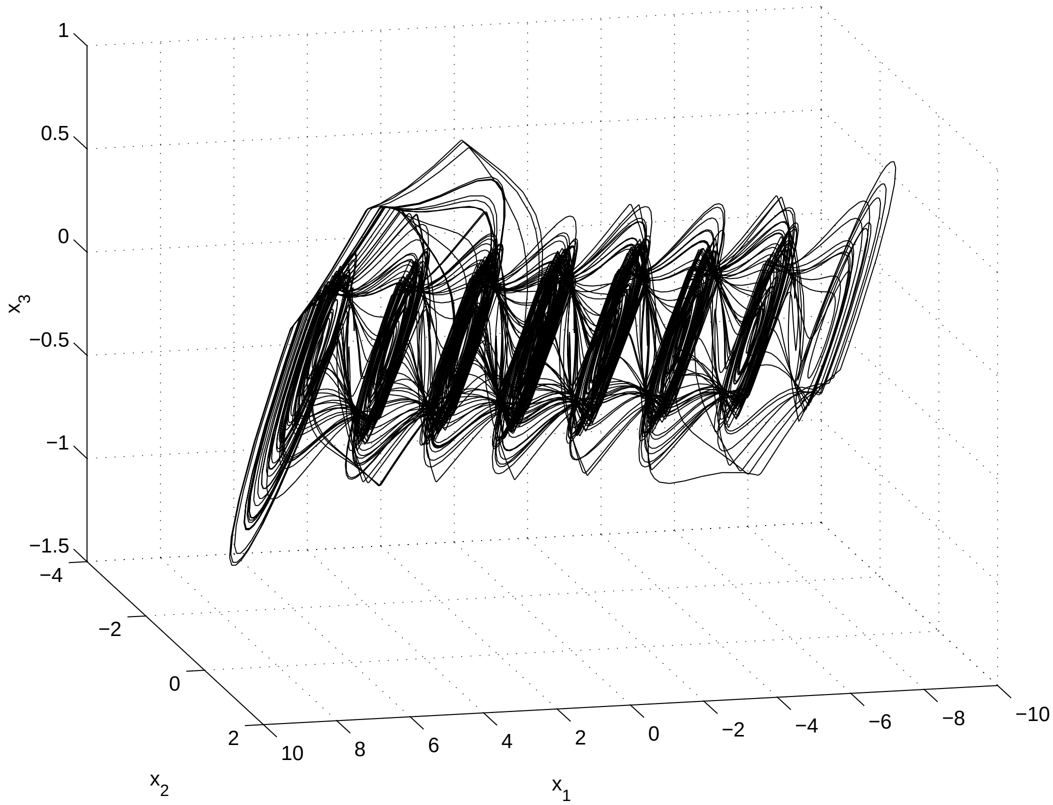
Figure captions:

Fig. 1 (Top) 8-scroll attractor ($N = 4$, $M = 4$, $a = 0.4$); (Bottom) nonlinearity $f(x_1)$.

Fig. 2 CFOA-based chaotic oscillator.

Fig. 3 $5-$scroll: Experimental result shown is (V_{R1}, V_{R3}) trajectory. $x = 1V/div.$, $y = 0.5V/div.$

Figure 1



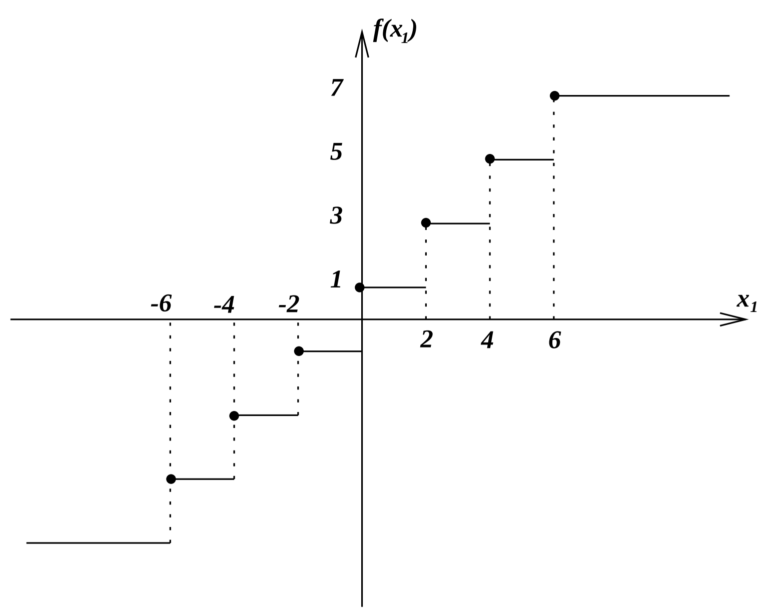
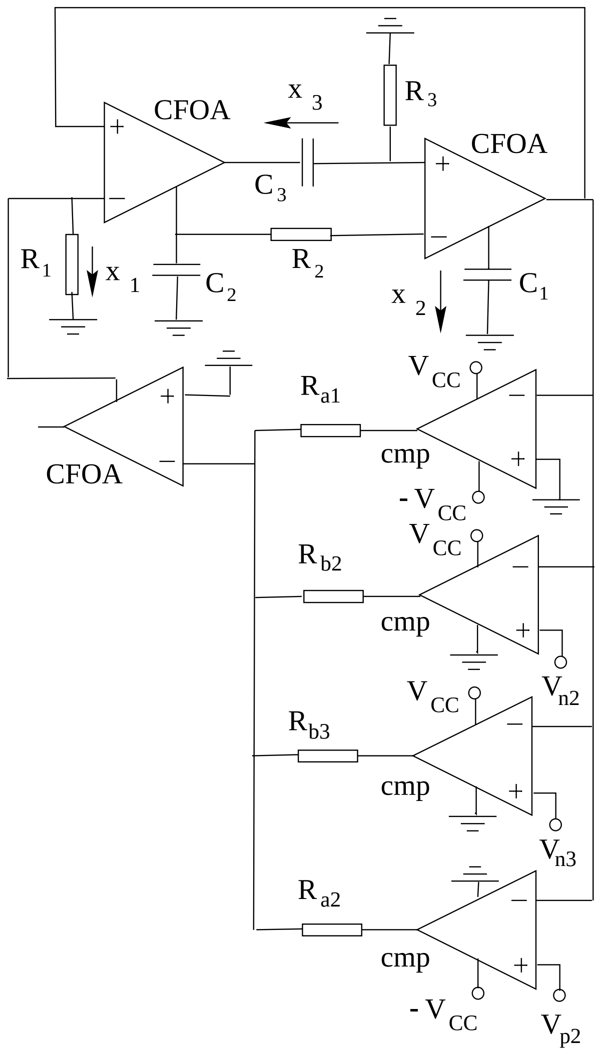


Figure 2


CFOA
x 3
R 3
CFOA
C 3
R 1
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V p2

Figure 3